

Drift optimization of high-rise buildings in earthquake zones

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SUMMARY

This paper presents a Drift Design Structural Model (DDSM) for the design optimization of high-rise buildings in seismic zones. The model is formulated as a Generalized Single Degree of Freedom System subjected to equivalent static seismic loadings. The model objectives are: (a) the minimization of the structure weight; (b) the minimization of the structure top drift; and (c) the uniform distribution of the inter-story drifts over the building height in order to minimize earthquake damage through the increase in plastic ductility. Seven high-rise buildings were analysed in order to validate the model, to illustrate its use and to demonstrate its capabilities in structural design optimization in earthquake zones. The results obtained show that the DDSM performed well and consequently can be of practical value to structural designers. Copyright © 2009 John Wiley & Sons, Ltd.

1. INTRODUCTION

In earthquake zones, the top displacement and the inter-story drifts of high-rise buildings must not exceed specified limits with respect to structure and story heights (Carpenter, 2004). Excessive lateral displacements and/or inter-story drifts may cause the failure of both structural and non-structural elements. The traditional trial-and-error design method, which is based on intuition and experience, is time consuming because high-rise buildings are complex and large scale in nature.

Mathematical optimizations provide methodologies to automate the structural design process. Further, one can achieve an optimum design solution out of numerous solutions on the basis of a selected criterion such as minimum weight or minimum cost. A number of articles have been published on the optimization of various kinds of structures with the majority focusing on the minimum weight design due to gravity loads. Only a small fraction of these articles dealt with the optimal drift design due to seismic loads. The published work on the optimal drift design of tall buildings (Park and Park, 1997; Park and Kwon, 2003; Chan, 2004; Chan and Wang, 2005; Lagaros and Papadrakakis, 2007) used structural optimization algorithms, which are based on sensitivity coefficients rather than practical optimizations, and require extensive computational requirements.

This paper presents a design optimization model for high-rise buildings under equivalent static seismic loadings based on the generalized single degree of freedom systems (Chopra, 2001). The proposed model minimizes the building weight and top drift while uniformly distributing inter-story drifts over the building height in order to increase the plastic ductility and to reduce earthquake damage. Hence, the design optimization model can be of practical value to structural designers.

2. RESEARCH SIGNIFICANCE

The design optimization of structure base due to gravity loads has been studied by a number of researchers. The publications on the optimal drift design due to seismic loads are relatively scarce. This paper presents a design optimization model for high-rise buildings under equivalent static seismic

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loadings based on the generalized single degree of freedom systems. The proposed model minimizes the building weight and top drift while uniformly distributing the inter-story drifts over the building height in order to increase the plastic ductility and to reduce earthquake damage. Hence, the design optimization model can be of practical value to structural designers.

3. MODEL FORMULATION

The primary purpose of this development stage is to formulate an optimization model that supports weight and drift minimization of high-rise buildings under earthquake loads. To this end, the present model is formulated in two major steps: (a) determining the major decision variables affecting the design of high-rise buildings under seismic loads; and (b) formulating the objective function for the weight and drift optimization of high-rise buildings in an optimization model.

Given a set of constant parameters, the optimization problem consists of finding the values of the design variables that simultaneously satisfy all of the design constraints and minimize the objective function.

3.1. Constant parameters

The constant parameters of the optimization problem include the following:

- Geometry: number of spans, span widths, member connectivity conditions and support conditions
- Loads: Uniform Building Code (UBC; International Conference of Building Officials, 1997) Static Equivalent Earthquake loads
- Material properties: steel and concrete moduli of elasticity, steel yield strength, concrete compressive strength, and concrete and steel unit weights

3.2. Design variables

The model is designed to consider all relevant decision variables that may have an impact on weight and drift optimization of high-rise buildings. This include the following member cross-sectional dimensions: (a) column depth h_C ; (b) column width b_C ; (c) beam depth h_B ; (d) beam width b_B ; (e) slab thickness t_S ; and (f) shear wall thickness t_W .

3.3. Objective function

The present optimization model is formulated to achieve the minimum weight and drift design for high-rise buildings under earthquake loads. Considering the building structure with N stories, which is shown in Figure 1, the design optimization objective function can be written as follows.

$$\text{Minimize Total Weight} = W = \sum_{i=1}^N \left[\sum_{j=1}^{N_C} WC_{ij} + \sum_{k=1}^{N_B} WB_{ik} + \sum_{l=1}^{N_S} WS_{il} + \sum_{m=1}^{N_W} WW_{im} \right] \quad (1)$$

where WC_{ij} , WB_{ik} , WS_{il} , WW_{im} = weight of column j , beam k , slab l and shear wall m at story i , respectively, and N_C , N_B , N_S , N_W = structure total number of columns, beams, slabs and walls, respectively.

The minimization of the objective function is subjected to the design code constraints, which are described briefly in the following section.

3.4. Design constraints

3.4.1. Drift constraints

The building top displacement δ_{op} must be less than or equal to the allowable drift value δ^U , which is prescribed by the design code. The constraint can be expressed as:

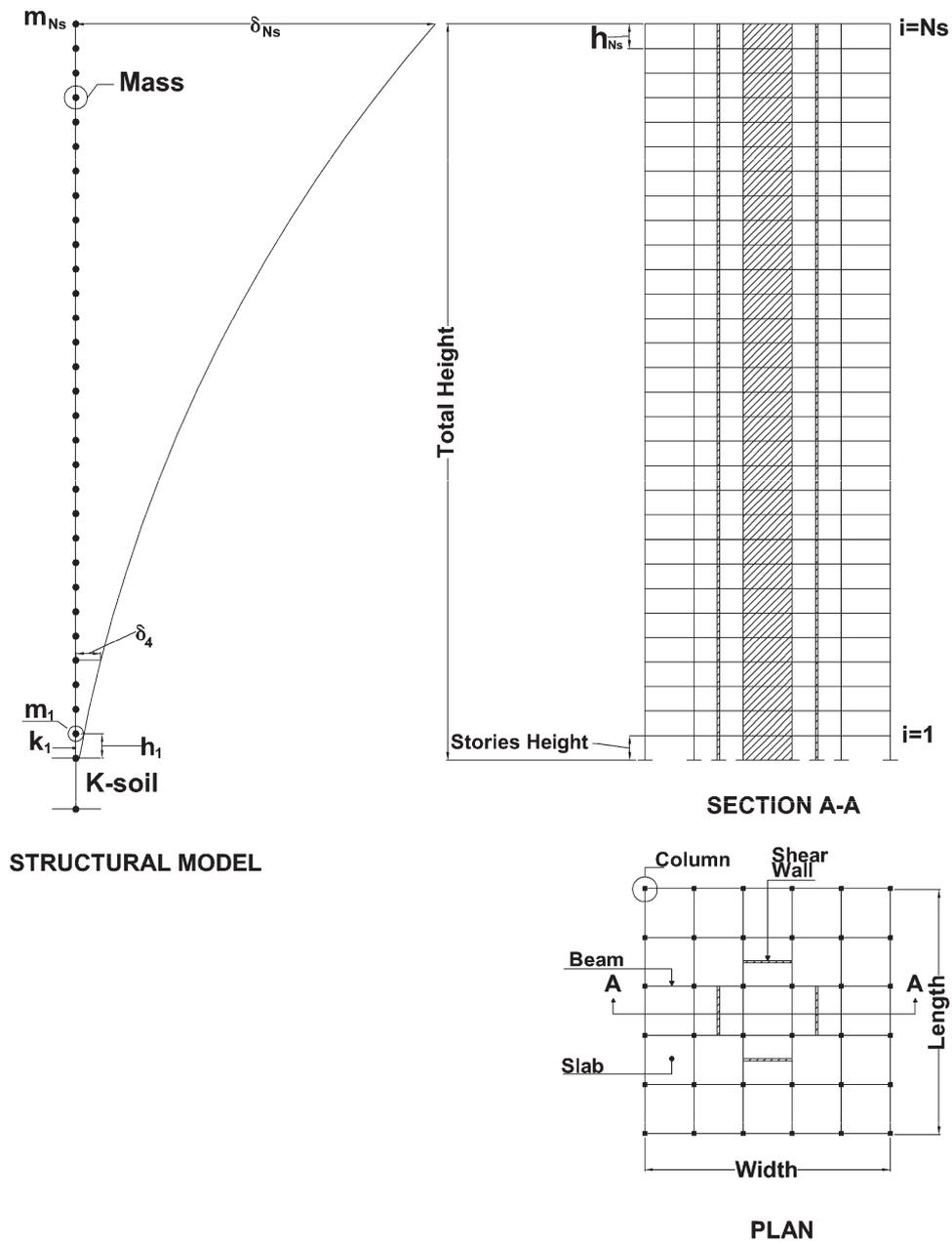


Figure 1. Building plan, elevation and structural model.

$$\frac{\delta_{top}}{H} \leq \delta^U \tag{2}$$

where H = building height.

The building drift/displacement δ_d between two consecutive floors must also be less than or equal to the allowable value δ_d^U . This constraint can be expressed as:

$$\frac{\delta_d \Psi_i}{h} \leq \delta_d^U \quad d = 1, 2, \dots, N_d \tag{3}$$

where h = building story height, and Ψ_i = shape vector.

3.4.2. Beam–column connection constraints

In seismic zones, design codes require that the moment of inertia of columns be larger than that of beams in order for the plastic hinges to form in beams rather than columns. This eliminates the risk of a total building collapse. This constraint can be expressed as:

$$IC_j \geq \alpha IB_k \quad (4)$$

where IC_j = column moment of inertia, IB_k = beam moment of inertia and α = design factor ≥ 1.2 .

3.4.3. Structural member size constraints

The structural member sizes must satisfy the following constraints:

$$h_C^L \leq h_C \leq h_C^U \quad \text{Column depth constraint} \quad (5)$$

$$b_C^L \leq b_C \leq b_C^U \quad \text{Column width constraint} \quad (6)$$

$$h_B^L \leq h_B \leq h_B^U \quad \text{Beam height constraint} \quad (7)$$

$$b_B^L \leq b_B \leq b_B^U \quad \text{Beam width constraint} \quad (8)$$

$$t_s^L \leq b_s \leq t_s^U \quad \text{Slab thickness constraint} \quad (9)$$

$$t_W^L \leq t_W \leq t_W^U \quad \text{Shear wall constraint} \quad (10)$$

where h_C^L and h_C^U = column depth lower and upper bounds, b_C^L and b_C^U = column width lower and upper bounds, h_B^L and h_B^U = beam height lower and upper bounds, b_B^L and b_B^U = beam width lower and upper bounds, t_s^L and t_s^U = slab thickness lower and upper bounds, and t_W^L and t_W^U = shear wall thickness lower and upper bounds.

4. MODEL IMPLEMENTATION

The generalized single degree of freedom system (GSDFS) equation of motion is given by the following equations:

$$\tilde{m}\ddot{z} + \tilde{k}z = -\tilde{L}\ddot{u}_g(t) \quad (11)$$

Rearranging Equation (11)

$$\tilde{m}\ddot{\delta} + \tilde{k}\delta = -\tilde{L}(\Psi\ddot{\delta})_g(t) \quad (12)$$

The generalized mass, stiffness and excitation are, respectively, given by the following equations:

$$\tilde{m} = \sum_i^N m_i (\Psi_i)^2 \quad (13)$$

$$\tilde{k} = \frac{\sum_i^N k_i (\Psi_i - \Psi_{i-1})^2 (k_{soil})}{\sum_i^N k_i (\Psi_i - \Psi_{i-1})^2 + (k_{soil})} \quad (14)$$

Table 1. Soil sub grade modulus (E_s).

Soil type (S)	Es (kN/m ³)
Rock (S1)	300000
Stiff soil (S2)	64 000–128 000
Stiff soil (S3)	32 000–80 000

$$\tilde{L}\ddot{u}_g(t) = \sum_i^N m_i \Psi_i \ddot{u}_g(t) \quad (15)$$

The story mass and stiffness are, respectively, given by the following equations:

$$m_i = \frac{W_i}{g} \quad (16)$$

$$k = \sum_i (k_C + k_B + k_W) \quad (17)$$

The soil stiffness is given by the following equation (Table 1).

$$k_{soil} = \text{Building area}(A) \times \text{Soil sub grade modulus}(E_s) \quad (18)$$

An optimization algorithm using MathCad Software (PTC, 2007) was employed herein to solve the optimization problem and to select the optimum member sizes that yield the minimum weight and top drift as well as uniform inter-story drifts. The design procedure is summarized in the flowchart shown in Figure 2. The design data such as member properties, ground acceleration, soil and material properties are used to compute weights and masses. The soil stiffness and the sizes of the columns, beams and shear walls are determined and used to determine the generalized mass, stiffness and excitation (\tilde{m} , \tilde{k} , \tilde{L}). Equation (12) is then used to compute the building top drift as well as inter-story drifts. Building member sizes and displacements are checked against Equations (2–10) to check the design constraints.

The GSDFS method allows the user to select the sizes of slabs, beams, columns and walls in each floor. The large number of members does not reduce the efficiency of the method since no extra computational time is needed. Moreover, no complicated mathematical operations are needed. This method determines moments and shear forces in addition to displacements. The GSDFS method is used for the initial design of the elements of high-rise structural systems, which leads to an accepted top drift (roof displacement), as well as allowable inter-story drift values.

5. EXAMPLES

The proposed optimization model was used for the optimum weight and drift design of seven high-rise buildings subjected to seismic loads. The limits of the top displacement and the inter-story drifts were, respectively, set equal to 0.005 times the total building height H and 0.02 times the story height h , respectively. The obtained top and inter-story displacements were compared with those obtained using a commercial 3-D finite element analysis software (Research Engineers (Europe) Limited, 2003). The buildings were subjected to the equivalent static loading prescribed by the UBC, which is defined by the following maximum base shear:

$$V = \frac{C_v I}{RT} W \quad (19)$$

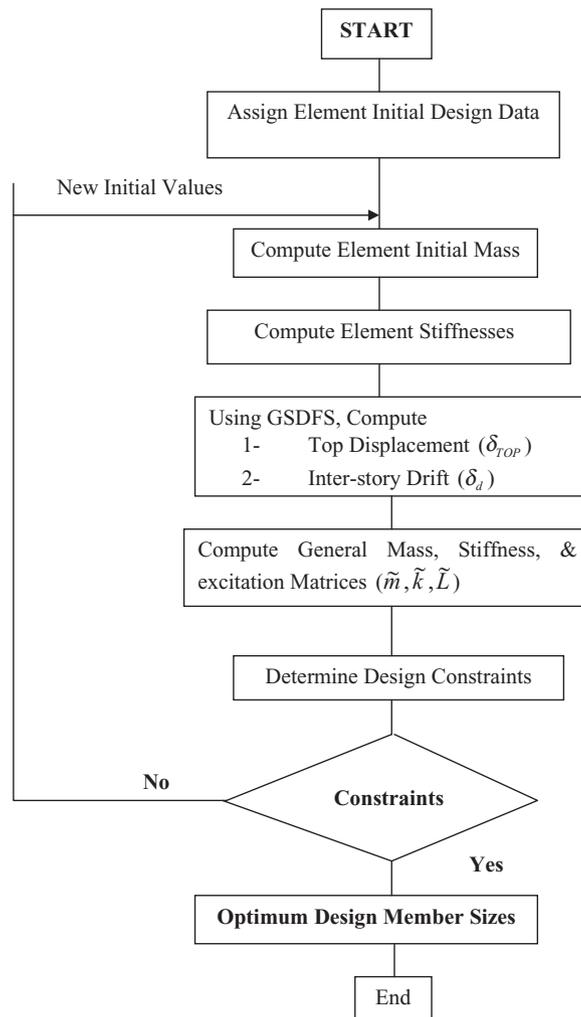


Figure 2. Flow chart of drift structural design model.

where C_V = seismic coefficient (UBC Table 16-R), I = building importance factor (UBC Table 16-K), R = numerical coefficient for ductility and strength (UBC Table 16-N) and T = elastic fundamental period of vibration, in seconds, of the structure in the direction under consideration. All of the structures considered were symmetrical and therefore no torsional effects were considered in the analysis. On the other hand, the seven structures had different material properties and were subjected to different earthquake loadings as shown in Tables 2 and 3.

5.1. Example 1

Two 30-storey concrete buildings, composed of shear walls, slabs, beams and columns were subjected to an earthquake acceleration of 1 g. The first building has a uniform story height of 3 m while the second building has non-uniform story heights of 3, 4 and 5 m as shown in Table 4 and Figure 3. The analysis using the proposed method yielded a maximum top drift of 0.438 m and a uniform inter-story drift of 0.015 m for the first building and a maximum top drift of 0.527 m and a uniform inter-story drift of 0.019 m for the second building. On the other hand, the 3-D finite element analysis yielded a top drift of 0.412 m and an inter-story drift ranging from 0.01 to 0.018 m for the first building and a top drift of 0.513 m and an inter-story drift ranging from 0.01 to 0.024 m for the second building. The results obtained are summarized in Table 5 and Figure 4. The results obtained using the proposed method are close to those obtained using the 3-D finite element software.

Table 2. Building loadings and material properties.

Example number	Acceleration (m/s ²)	E _{concrete} (kN/m ²)*	E _{steel} (kN/m ²) [†]	γ _{concrete} (kN/m ³) [‡]	γ _{steel} (kN/m ³) [§]	Story height
1	1 g	2.74 10 ⁷	–	25	–	Uniform
2	0.75 g	2.74 10 ⁷	–	25	–	Non-uniform
3	0.4 g	3.16 10 ⁷	–	25	–	Uniform
4	0.2 g	3.16 10 ⁷	–	25	–	Non-uniform
5	0.2 g	2.74 10 ⁷	2.05 10 ⁸	25	76.8	Non-uniform
6	0.075 g	3.54 10 ⁷	2.05 10 ⁸	25	76.8	Uniform
7	0.075 g	3.54 10 ⁷	2.05 10 ⁸	25	76.8	Non-uniform

* Concrete modulus of elasticity.

[†] Steel modulus of elasticity.[‡] Concrete unit weight.[§] Steel unit weight.

g, gravitational acceleration.

Table 3. Structural elements dimension range.

Structural element dimensions	Minimum value (m)	Maximum value (m)
Column depth, h _C	0.3	1.20
Column width, b _C	0.3	1.20
Beam depth, h _B	0.3	0.80
Beam width, b _B	0.3	0.40
Slab thickness, t _s	0.2	0.35
Shear wall thickness, t _w	0.3	0.40

Table 4. Thirty-storey building member dimensions.

Optimum values of design variables	Uniform story height (m)	Non-uniform story height (m)
Column depth, h _C	0.8	0.9
Column width, b _C	0.8	0.9
Beam depth, h _B	0.5	0.5
Beam width, b _B	0.2	0.2
Slab thickness, t _s	0.2	0.2
Shear wall thickness, t _w	0.3	0.4

5.2. Example 2

Two 50-storey buildings, composed of shear walls, slabs, beams and columns were subjected to an earthquake acceleration of 0.4 g. The first building has a uniform story height of 3 m while the second building has non-uniform story heights of 3 4 and 5 m as shown in Table 6 and Figure 5. The analysis using the proposed method yielded a maximum top drift of 0.710 m and a uniform inter-story drift of 0.014 m for the first building and a maximum top drift of 0.872 m and a uniform inter-story drift of 0.019 m for the second building. The 3-D finite element analysis resulted in a top drift of 0.710 m and an inter-story drift ranging from 0.010 to 0.018 m for the first building and a top drift of 0.718 m and an inter-story drift ranging from 0.010 to 0.022 m for the second building. The results obtained are summarized in Table 7 and Figure 6. The results obtained using the proposed method are close to those obtained using the 3-D finite element software.

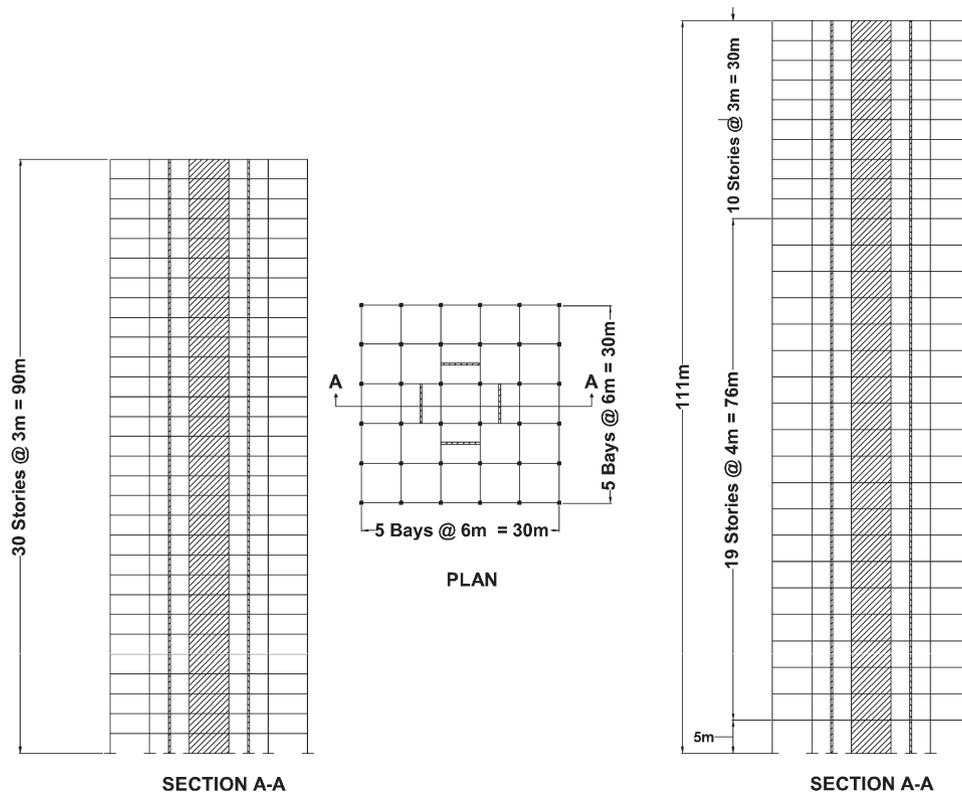


Figure 3. Thirty-storey building plan and elevation for uniform and non-uniform height.

Table 5. Thirty-storey concrete building drifts.

Analysis results	Analysis method	Soil type	Uniform story height	Non-uniform story height
Roof displacement, (m)	GSDFS	Soil S1	0.423	0.519
		Soil S2	0.426	0.520
		Soil S3	0.438	0.527
Inter-story displacement, (m)	Finite element Maximum	Soil S1, S2 and S3	0.412	0.513
		Soil S1, S2 and S3	0.450	0.555
	GSDFS	Soil S1	0.014	0.019
		Soil S2	0.014	0.019
		Soil S3	0.015	0.019
	Weight, kips (kN)	Finite element Maximum	Soil S1, S2 and S3	0.010–0.018
		Soil S1, S2 and S3	0.060	0.100
			230000	265900

5.3. Example 3

A 50-storey building, composed of concrete shear walls and a steel frame was subjected to 0.2 g earthquake loading. It has non-uniform story heights of 3, 4 and 5 m as shown in Table 8. The analysis using the proposed methods yielded a maximum top drift of 0.872 m and a uniform inter-story drift of 0.019 m. The 3-D finite element analysis gave a top drift of 0.768 m and an inter-story drift ranging from 0.010 to 0.022 m. The results obtained are summarized in Table 9. The results obtained using the proposed method are close to those obtained using the 3-D finite element software.

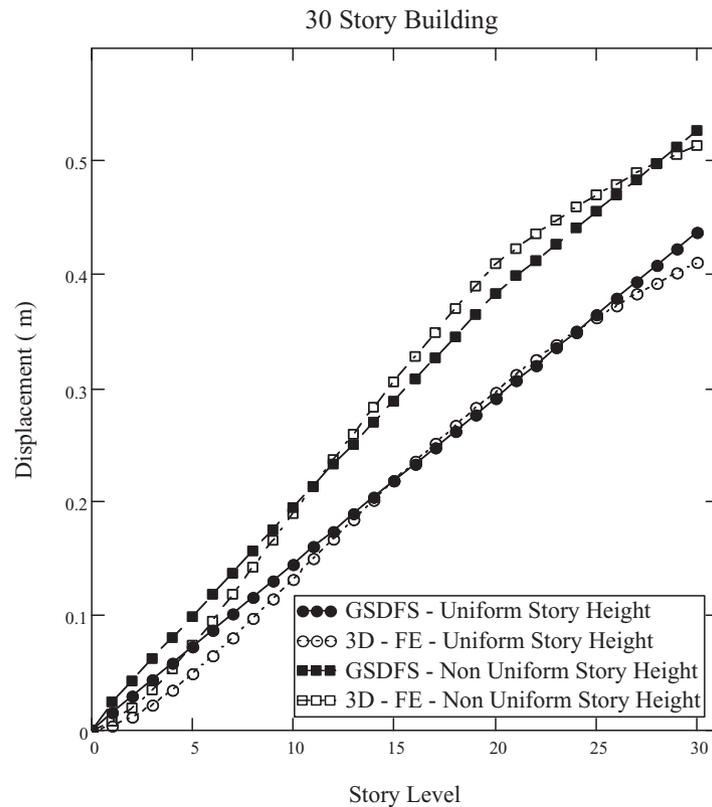


Figure 4. GSDFS optimum model drift versus 3-D finite element drift.

Table 6. Fifty-storey building member dimensions.

Optimum values of design variables	Uniform story height (m)	Non-uniform story height (m)
Column depth, h_c	1.05	0.85
Column width, b_c	1.05	0.85
Beam depth, h_B	0.50	0.50
Beam width, b_B	0.20	0.20
Slab thickness, t_s	0.20	0.20
Shear wall thickness, t_w	0.30	0.30

5.4. Example 4

Two 100-storey buildings composed of concrete shear walls and slabs and a steel frame were subjected to 0.075 g earthquake loading. The first building has a uniform story height of 3 m while the second has non-uniform story heights of 3, 4 and 5 m as shown in Table 10 and Figure 7. The analysis using the proposed method yielded a maximum top drift of 1.418 m and a uniform inter-story drift of 0.014 m for the first building and a maximum top drift of 1.797 m and a uniform inter-story drift of 0.019 m for the second building. The 3-D finite element analysis gave a top drift of 1.21 m and an inter-story drift ranging from 0.010 to 0.014 m and a top drift of 1.624 m and an inter-story drift ranging from 0.010 to 0.022 m for the second building. The results obtained are summarized in Table 11 and Figure 8. The results obtained using the proposed method are close to those obtained using the 3-D finite element software.

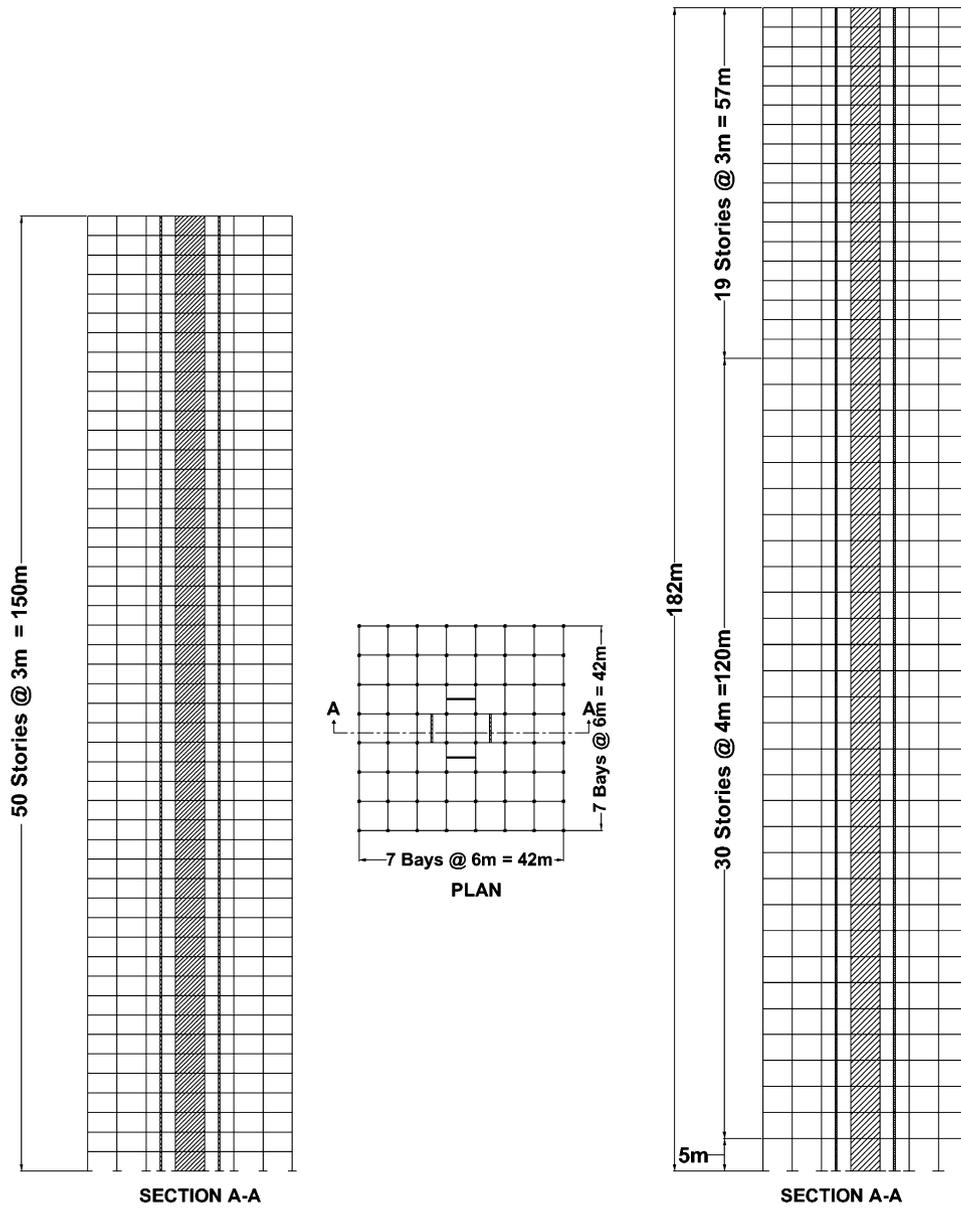


Figure 5. Fifty-storey building plan and elevation for uniform and non-uniform height.

In the example, the weight of the concrete structure was determined using the following equation:

$$\begin{aligned}
 W(h_C, b_C, h_B, b_B, t_S, t_W) &= \sum_i \left(\sum_j W_C + \sum_k W_B + \sum_z W_S + \sum_w W_W \right) \\
 &= \sum_i \left(\sum_j h_{C_j} \cdot b_{C_j} \cdot h_i \cdot \gamma_C + \sum_k h_{B_k} \cdot b_{B_k} \cdot L_k \cdot \gamma_B \right. \\
 &\quad \left. + \sum_z t_{S_z} \cdot A_{S_z} \cdot \gamma_S + \sum_w t_{W_w} \cdot D_{W_w} \cdot h_i \cdot \gamma_W \right) \tag{20}
 \end{aligned}$$

where $\gamma_C, \gamma_B, \gamma_S, \gamma_W$ = unit weights of columns, beams, slabs and shear walls, respectively, A_s = slab area (Panel area), D_w = shear wall depth and L_k = beam length.

Table 7. Fifty-storey building drifts.

Analysis results	Analysis method	Soil type	Uniform story height	Non-uniform story height
Roof displacement, (m)	GSDFS	Soil S1	0.708	0.872
		Soil S2	0.708	0.872
		Soil S3	0.710	0.872
Inter-story displacement, (m)	Finite element	Soil S1, S2 and S3	0.710	0.513
		Maximum	0.750	0.718
	GSDFS	Soil S1	0.014	0.019
		Soil S2	0.014	0.019
		Soil S3	0.014	0.019
Weight, kips (kN)	Finite element	Soil S1, S2 and S3	0.010–0.018	0.010–0.022
		Maximum	0.060	0.100
			816000	772500

Table 8. Fifty-storey mixed structure member dimensions.

Optimum values of design variables	Non-uniform story height (m)
Column steel W-shape	W _{1000×883}
Column depth, h_c	0.850
Column width, b_c	0.850
Steel column web depth, h_{c1}	0.928
Steel column web thickness, a_c	0.045
Beam steel W-shape	W _{360×64}
Beam depth, h_b	0.347
Beam width, b_b	0.203
Steel beam web depth, h_{b1}	0.347
Steel beam web thickness, a_b	0.008
Slab thickness, t_s	0.200
Shear wall thickness, t_w	0.300

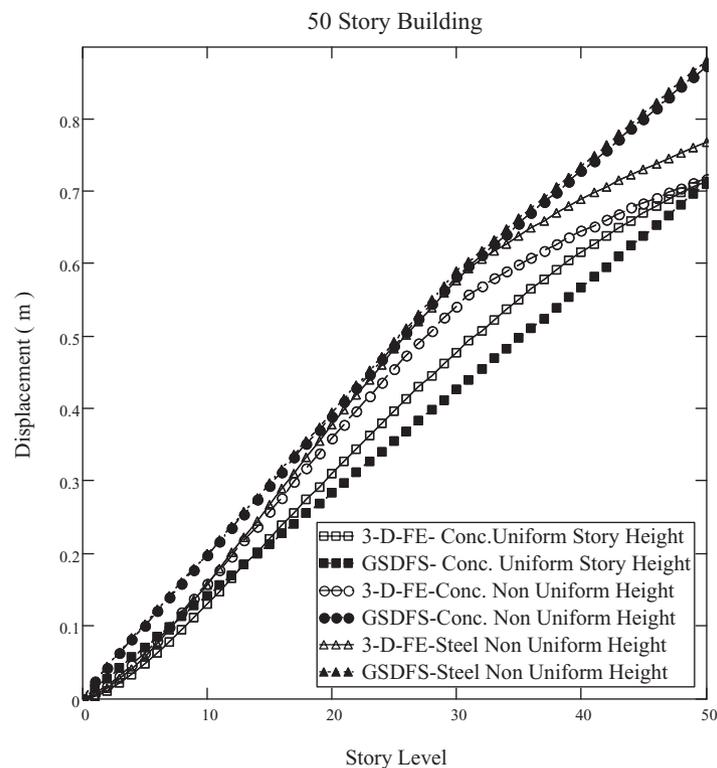


Figure 6. GSDFS optimum model drift versus 3-D finite element drift.

Table 9. Fifty-storey mixed structure drifts.

Analysis results	Analysis method	Soil type	Non-uniform story height
Roof displacement, (m)	GSDFS	Soil S1	0.873
		Soil S2	0.874
		Soil S3	0.877
	Finite element	Soil S1, S2 and S3	0.768
		Maximum	0.905
Inter-story displacement, (m)	GSDFS	Soil S1	0.019
		Soil S2	0.019
		Soil S3	0.019
	Finite element	Soil S1, S2 and S3	0.010–0.022
		Maximum	0.060
Steel weight, kips (kN)			119 800
Concrete weight, kips (kN)			473 600

Table 10. One hundred-storey building member dimensions.

Optimum values of design variables	Uniform story height (m)	Non-uniform story height (m)
Column steel W-shape	W _{1000×415}	W _{1000×383}
Column depth, h_c	1.008	1.075
Column width, b_c	0.402	0.419
Steel column web depth, h_{c1}	0.928	0.928
Steel column web thickness, a_c	0.021	0.041
Beam steel W-shape	W _{360×64}	W _{360×64}
Beam depth, h_b	0.347	0.347
Beam width, b_b	0.203	0.203
Steel beam web depth, h_{b1}	0.347	0.347
Steel beam web thickness, a_b	0.008	0.008
Slab thickness, t_s	0.200	0.200
Shear wall thickness, t_w	0.300	0.300

Table 11. One hundred-storey building drifts.

Analysis results	Analysis method	Soil type	Non-uniform story height
Roof displacement, (m)	GSDFS	Soil S1	1.418
		Soil S2	1.418
		Soil S3	1.418
	Finite element	Soil S1, S2 and S3	1.210
		Maximum	1.500
Inter-story displacement, (m)	GSDFS	Soil S1	0.014
		Soil S2	0.014
		Soil S3	0.014
	Finite element	Soil S1, S2 and S3	0.010–0.014
		Maximum	0.060
Steel shape weight, kips (kN)			89 460
Concrete weight, kips (kN)			936 000

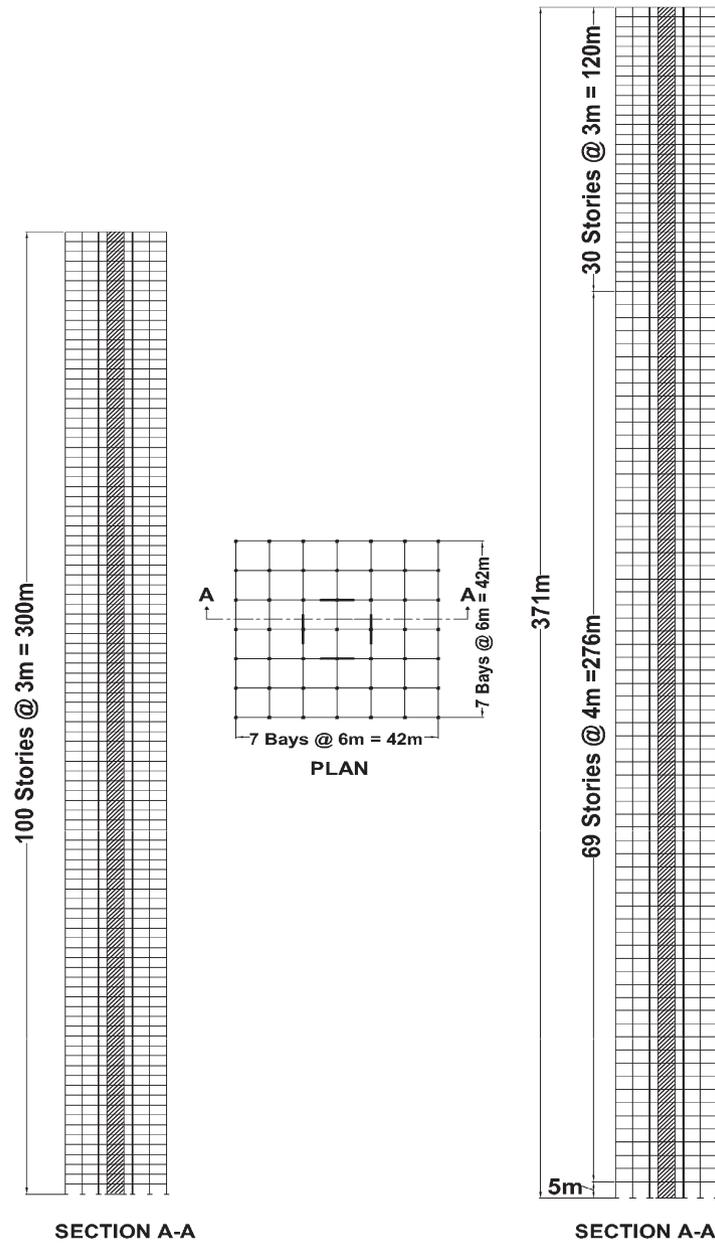


Figure 7. One hundred-storey building plan and elevation for uniform and non-uniform height.

The weight of concrete shear walls and slabs and the steel frame is given by the following equation:

$$\begin{aligned}
 W(h_C, b_C, h_B, b_B, t_S, t_W) &= \sum_i \left(\sum_j W_C + \sum_k W_B + \sum_z W_S + \sum_w W_W \right) \\
 &= \sum_i \left(\sum_j ((ac \cdot hc1) + (h_{Cj} - hc1) \cdot b_{Cj}) \cdot h_i \cdot \gamma_C \right. \\
 &\quad + \sum_k ((ab \cdot hb1) + (h_{Bk} - hb1) \cdot b_{Bk}) \cdot L_k \cdot \gamma_B \\
 &\quad \left. + \sum_z t_{S_z} \cdot A_{S_z} \cdot \gamma_S + \sum_W t_{W_w} \cdot D_{W_w} \cdot h_i \cdot \gamma_W \right) \quad (21)
 \end{aligned}$$

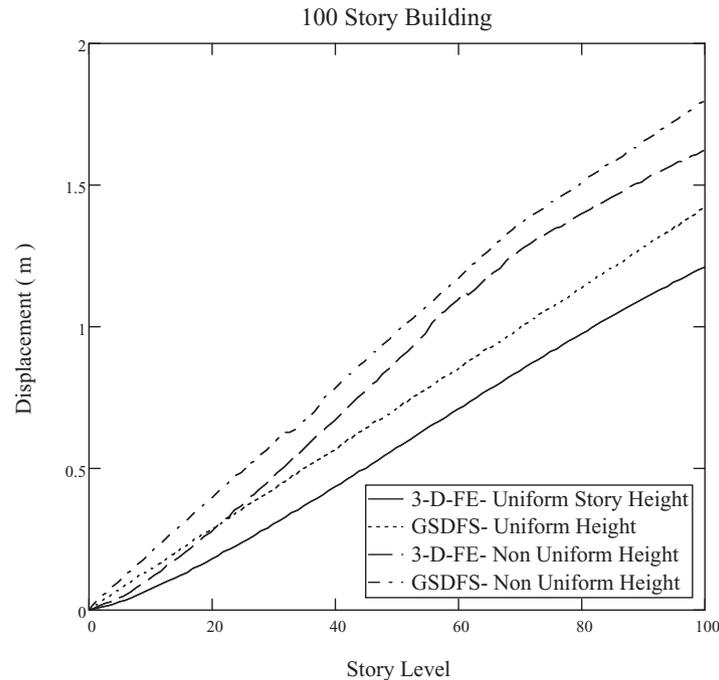


Figure 8. GSDFS optimum model drift versus 3-D finite element drift.

where a_c and a_b = web thickness of steel columns and beams, respectively and h_{c1} and h_{b1} = web depth of steel columns and beams, respectively.

6. CONCLUSIONS

The paper presented a computational Drift Design Structural Model for the optimum drift design of structural elements subjected to equivalent static seismic loading. The model formulation includes the lateral top displacement and inter-story drifts of a building. The model has demonstrated through the example, structures analyzed that the top lateral displacement and the inter-story drifts of tall buildings can be controlled to produce the optimum design sections of the building structural elements. It can be concluded that the proposed model has the following features:

- (1) Yields faster optimum design of structural elements as compared with the time consuming trial-and-error design
- (2) Computes top displacement and inter-story drifts using the weight of slabs, beams, columns and shear walls as well as the stiffness of columns, beams, shear walls and soil–structure interaction
- (3) Yields structure top displacements and inter-story drifts that compare well with 3-D finite element results
- (4) Performs very well with concrete and mixed concrete–steel structures. Since the presented model uses in its formulation material unit weight and modulus of elasticity, it can easily optimize the structural elements of any building regardless of its material.
- (5) Takes into account the weight of all structural members in weight calculation and takes into account of the stiffness of the columns, beams, shear walls and soil structure interaction in the stiffness calculation of the whole structure

The proposed model is suitable for the optimal design of structure elements because it minimizes the structural weight and leads to an acceptable uniform plastic ductility of the building.

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REFERENCES

- Carpenter LD. 2004. High-rise building cladding drift accommodation. *The Structural Design of Tall and Special Buildings* **13**: 439–456.
- Park H, Park CL. 1997. Drift control of high-rise building with unit load method. *The Structural Design of Tall and Special Buildings* **6**: 23–35.
- Park HS, Kwon JH. 2003. Optimal drift design model for multi-story buildings subjected to dynamic lateral forces. *The Structural Design of Tall and Special Buildings* **12**: 317–333.
- Chan C-M. 2004. Advances in structural optimization of tall buildings in Hong Kong. In *Proceedings of Third China-Japan-Korea Joint Symposium on Optimization of Structural and Mechanical Systems*, Kanazawa, Japan.
- Chan C-M, Wang Q. 2005. Optimal drift design of tall reinforced concrete buildings with non-linear cracking effects. *The Structural Design of Tall and Special Buildings* **14**: 331–351.
- Lagaros ND, Papadrakakis M. 2007. Seismic design of RC structures: a critical assessment in the framework of multi-objective optimization. *Earthquake Engineering & Structural Dynamics* **36**: 1623–1639.
- Chopra AK. 2001. *Dynamics of Structures-Theory and Application to Earthquake Engineering* (2nd edn). Prentice Hall, Inc.: Upper Saddle River, NJ.
- International Conference of Building Officials. 1997. Uniform Building Code. Whittier, CA, USA.
- PTC. 2007. Mathcad 13. PTC Corporate Headquarters, 140 Kendrick Street, Needham, MA 02494, USA.
- Research Engineers (Europe) Limited. 2003. Staad Pro 2003. Draycott House, Almondsbury Business Center, Bristol, UK.