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ORIGINAL RESEARCH

Reliability and flexural behavior of triangular and T-reinforced concrete beams

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Abstract The paper studied the behavior of reinforced concrete triangular and T-beams. Three reinforced concrete beams were tested experimentally and analyzed analytically using the finite element method. Their reliability was also assessed using the reliability index approach. The results showed that the finite element vertical displacements compared well with those obtained experimentally. They also showed that the vertical displacements obtained using the finite element method were larger than those obtained experimentally. This is a strong indication that the finite element results were conservative and reliable. The results showed that the triangular beams exhibited higher ductility at failure than did the T-beam. The plastic deformations at failure of the triangular beams were higher than that of the T-beam. This is a strong indication of the higher ductility of the triangular beams compared to the T-beam. Triangular beams exhibited smaller cracks than did T-beams for equal areas of steel and concrete. The design moment strengths M_c computed using the American Concrete Institute (ACI) design formulation were safe and close to those computed using experimental results. The experimental results validated the reliability analysis results, which stated that the triangular beams are more reliable than T-beams for equal areas of steel and concrete.

Keywords Concrete beams · Experimental testing · Finite element · Nonlinear analysis · Reliability index

Introduction

The majority of structures built worldwide are made of reinforced concrete. Most of these structures use beams as a structural elements to resist applied loads. The reliability and response of these structural components were studied using the reliability index β and finite element analysis, respectively (Saifullah et al. 2011; Vecchio and Shim 2004). The reliability index β measures the level of reliability of the beams based on their response to applied loads and according to their design codes. The reliability index chart is very useful for determining the beam strength capacity for a desired level of reliability (Al-Ansari 2013a). The behavior of reinforced concrete structural elements was also studied using experimental testing. Concrete beams of different sections were analyzed for safety, stability, deformation, and crack formation based on ACI Ultimate Design Method [Lu et al. 1994; Borse and Dubey 2013; American Concrete Institute (ACI) 2008; McCormac and Brown 2009]. Experimental testing is time consuming and costly while finite element analysis is faster and less expensive. Finite element models have been developed for reinforced concrete beams to study their response at various load stages (Nahvi and Jabbari 2005). The objective of this paper is to study the flexural behavior of triangular and T-reinforced concrete beams subjected to center point loadings. The experimental load–deflection results were compared with those obtained using a non-linear finite element analysis (Bentley System Inc. 2009). The reliability of the beams was also assessed using the reliability index approach.

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Reliability formulation

A beam fails when its resistance is less than the action caused by the applied loads. The beam resistance and action are computed using the design strength M_c and the external bending moment M_e, respectively. Figure 1 shows the compressive and tensile cross sectional areas for and triangular and T-beams.

The beam limit state function is given by the following equation:

$$G(A_s,f'_c,f_y,M_e) = M_c - M_e \tag{1}$$

where M_c = design strength, M_e = external bending moment, A_s = tensile steel area, f_y = reinforcing steel yield strength, and f'_c = concrete compressive strength.

The triangular beam limit state function is given by the following equation:

$$G(A_s,f'_c,f_y,M_e) = \phi\mu_{A_s}\mu_{f_y}\left(d - \frac{2}{3}\sqrt{\frac{\frac{\mu_{A_s}\mu_{f_y}}{0.85\mu_{f'_c}}}{0.5\frac{b}{h}}}\right) - \mu_{M_e} \tag{2}$$

where ϕ = bending reduction factor, b = beam width, d = beam effective depth, h = beam depth, μ_{f_y} = mean value of f_y; $\mu_{f'_c}$ = mean value of f'_c, μ_{A_s} = mean value of A_s, and μ_{M_e} = mean value of M_e.

Because of its nonlinearity, the limit state function is linearized using the Taylor series expansion about the mean value using the following equation (Nowak and Collins 2013):

$$\begin{aligned} G(A_s,f'_c,f_y,M_e) = {} & \left(\phi\mu_{A_s}\mu_{f_y}\left(d - \frac{2}{3}\sqrt{\frac{\frac{\mu_{A_s}\mu_{f_y}}{0.85\mu_{f'_c}}}{0.5\frac{b}{h}}}\right) - \mu_{M_e}\right) \\ & + (A_s - \mu_{A_s})\frac{\mathrm{d}G}{\mathrm{d}A_s} + (f_y - \mu_{f_y})\frac{dG}{d_{f_y}} + (f'_c - \mu_{f'_c})\frac{\mathrm{d}G}{\mathrm{d}f'_c} \\ & + (f'_c - \mu_{f'_c})\frac{\mathrm{d}G}{\mathrm{d}f'_c} \end{aligned} \tag{3}$$

The reliability index β of the linear function is given by the following equation:

$$\beta = \frac{G(A_s,f'_c,f_y,M_e)}{\sqrt{(\sigma_{A_s}a_1)^2+\sqrt{(\sigma_{f_y}a_2)^2}+\sqrt{(\sigma_{f'_c}a_3)^2}+\sqrt{(\sigma_{M_e}a_4)^2}}} \tag{4}$$

where σ_{A_s} = standard deviation of A_s, σ_{f_y} = standard deviation of f_y, $\sigma_{f'_c}$ = standard deviation of f'_c; and σ_{M_e} = standard deviation of M_e. The parameters a_1, a_2, a_3, and a_4 are given by the following equations, respectively:

$$a_1 = \frac{\partial G}{\partial A_s}\left(\phi A_s f_y\left(d - \frac{2}{3}\sqrt{\frac{\frac{A_s f_y}{0.85f'_c}}{0.5\frac{b}{h}}}\right) - M_e\right) \tag{5}$$

$$a_2 = \frac{\partial G}{\partial f_y}\left(\phi A_s f_y\left(d - \frac{2}{3}\sqrt{\frac{\frac{As f_y}{0.85f'_c}}{0.5\frac{b}{h}}}\right) - M_e\right) \tag{6}$$

$$a_3 = \frac{\partial G}{\partial f'_c}\left(\phi A_s f_y\left(d - \frac{2}{3}\sqrt{\frac{\frac{A_s f_y}{0.85f'_c}}{0.5\frac{b}{h}}}\right) - M_e\right) \tag{7}$$

$$a_4 = \frac{\partial G}{\partial M_e}\left(\phi A_s f_y\left(d - \frac{2}{3}\sqrt{\frac{\frac{A_s f_y}{0.85f'_c}}{0.5\frac{b}{h}}}\right) - M_e\right) \tag{8}$$

The standard deviation σ is equal to the product of the mean value μ and the coefficient of variation V. The formulation estimates the reliability index β of triangular beams when subjected to flexural loads, based on their resistance to applied loads (Table 1; Fig. 2).

The limit state function of a T-beam with its neutral axis lying in the flange is given by the following equation:

$$G(A_s,f'_c,f_y,M_e) = \phi\mu_{A_s}\mu_{f_y}\left(d - \frac{1}{2}\frac{\mu_{A_s}\mu_{f_y}}{0.85\ \mathrm{b}\ \mu_{f'_c}}\right) - \mu_{M_e} \tag{9}$$

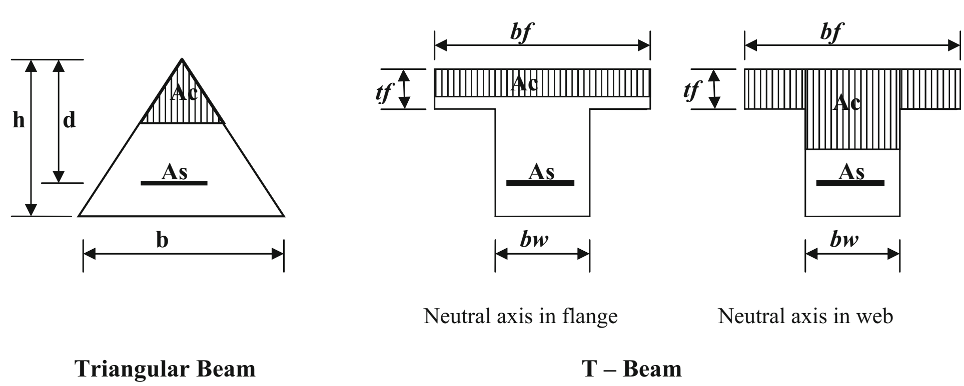

Fig. 1 Compressive and tensile section areas for T- and Triangular beams

Table 1 Triangular beam analysis results

M_e (kN m)	Beam data						M_c (kN m)	β	Safety percentage (%)
	f_c (MPa)	f_y (MPa)	b (mm)	h (mm)	d (mm)	A_s (mm^2)			
17	420	30	200	400	300	250	20	1.0	15
184	420	30	300	950	800	1050	230	1.5	20
390	420	30	400	1300	1100	1700	524	2.0	26
640	420	30	400	1600	1300	2100	753	1.0	15
890	420	30	600	1600	1400	3000	1200	2.0	26

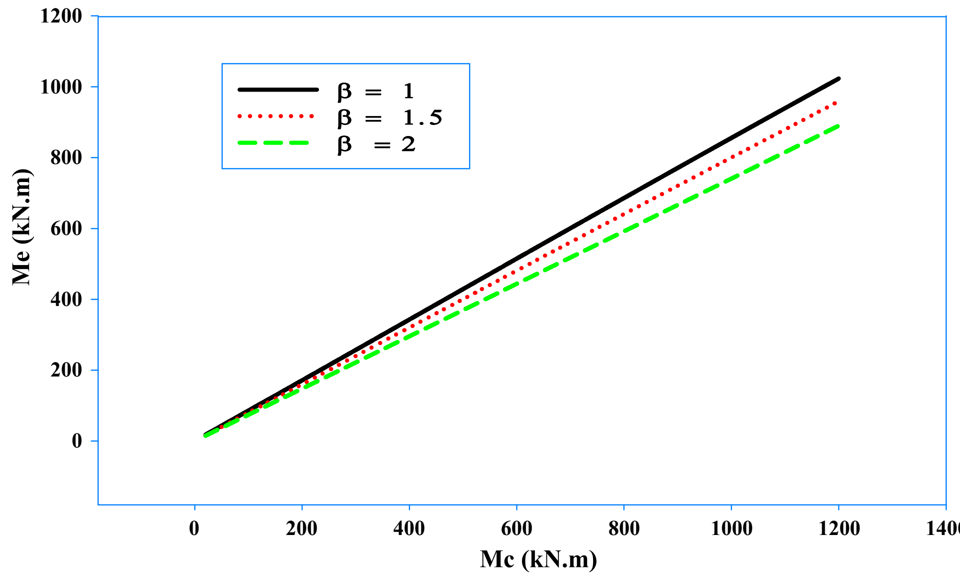

Fig. 2 Triangular beam reliability index

Table 2 T-beam analysis results (neutral axis in flange)

M_e (kN m)	Beam data							M_c (kN m)	β	Safety percentage (%)
	f_c (MPa)	f_y (MPa)	b_w (mm)	b_f (mm)	t_f (mm)	d (mm)	A_s (mm^2)			
20	30	420	200	300	100	300	250	27.7	2.0	28
212	30	420	200	450	100	750	900	250	1.0	15
406	30	420	200	450	100	1000	1400	516	1.5	21
820	30	420	200	450	100	1400	2200	1131	2.0	27
1230	30	420	200	500	100	1600	2450	1440	1.0	15

After function linearization and constant determination, the reliability index β is obtained using the following equation:

$$\beta = \frac{F(A_s, f_c', f_y, M_e)}{\sqrt{(\sigma_{A_s} a_1)^2} + \sqrt{(\sigma f_y a_2)^2} + \sqrt{(\sigma_{f_c'} a_3)^2} + \sqrt{(\sigma_{M_e} a_4)^2}} \quad (10)$$

The formulation allows the estimation of the reliability index β of a T-beam with the neutral axis in the flange when subjected to flexural loads (Table 2; Fig. 3).

The limit state function of a T-beam with its neutral axis lying in the web is given by the following equation:

$$W(A_s, f_c', f_y, M_e) = \phi\, \mu_{A_s}\, \mu_{f_y} (d - y) - \mu_{M_e} \quad (11)$$

where A_f = flange crosssectional area, A_c = compressive crosssectional area, t_f = flange thickness, b_w = web width. The location of the neutral axis y is given by the following equation:

$$y = \frac{A_f \frac{t_f}{2} + (A_c - A_f)(0.5 \frac{A_c - A_f}{b_w} + t_f)}{A_c} \quad (12)$$

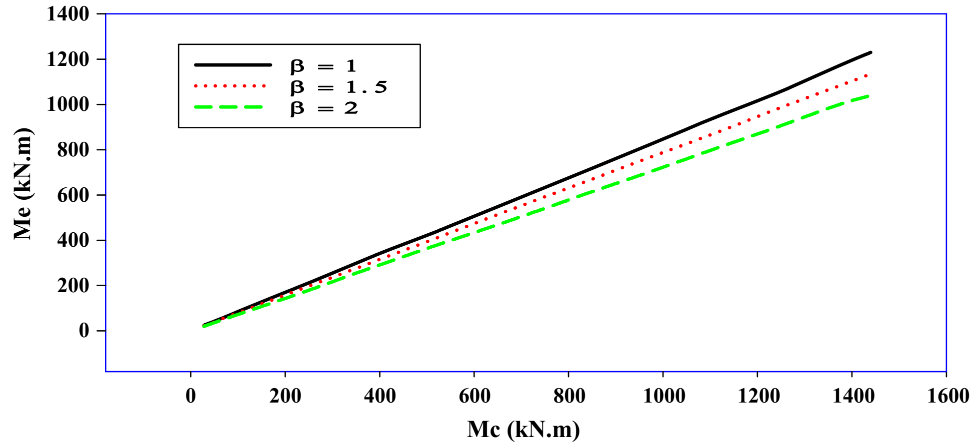

Fig. 3 Reliability index of T-beams with neutral axis in the flange

Table 3 T-beam analysis results (neutral axis in web)

M_e (kN m)	Beam data							M_c (kN m)	β	Safety percentage (%)
	f_c (MPa)	f_y (MPa)	b_w (mm)	b_f (mm)	t_f (mm)	d (mm)	A_s (mm^2)			
218	30	420	250	300	100	400	2000	261	1.0	16
415	30	420	300	400	100	600	2600	537	1.5	23
626	30	420	300	500	100	800	3100	878	2.0	29
1350	30	420	400	600	100	1200	3700	1607	1.0	16
1584	30	420	400	600	100	1500	3700	2027	1.5	22
1810	30	420	500	700	100	1600	4300	2518	2.0	28

After function linearization and constant determination, the reliability index β is obtained using the following equation:

$$\beta = \frac{W(A_s, f_c', f_y, M_e)}{\sqrt{(\sigma_{A_s} a_1)^2 + \sqrt{(\sigma_{f_y} a_2)^2} + \sqrt{(\sigma_{f_c'} a_3)^2} + \sqrt{(\sigma_{M_e} a_4)^2}}} \quad (13)$$

The reliability index β is calculated for a T-beam with its neutral axis in the web (Table 3; Fig. 4).

Experimental testing

Three reinforced concrete beams (two triangular beams and one T-beam) were tested at Qatar University to study their behavior under applied center loads (Instron 2003; Al-Ansari 2013b). The Instron HDX150 machine was used in the testing as shown in Fig. 5. The equipment has a 1500-kN-load-capacity testing in bending, compression, and shear. Figure 6 shows the schematic representation of the test setup. The three beams, which had a length of 2 meters, were prepared using a 33-MPa-compressive-strength concrete material. They were reinforced with two tensile T-12 steel bars. The shear reinforcement consisted of @8 stirrups spaced at a distance of 200 mm. The reinforcing steel bars had yield strength of 550 MPa. The beam cross sectional dimensions are summarized in Table 4. The concrete mix design is summarized in Table 5.

The testing machine provides an output data set that includes time, flexural load, stress and strain, and displacement. Table 6 shows a sample of the testing machine output set.

The Triangular I, Triangular II, and T-beams collapsed under concentrated loads of 34, 45, and 41 kN, respectively. Figure 7 shows the crack pattern and deformed shape of the beams after collapse.

The deflection of the beams was obtained from the output that was provided by the testing machine. Table 7 summarizes the failure load and deflection at collapse for the three beams. On the other hand, Fig. 8 shows the load–deflection response for the three beams.

The results show that the triangular beams exhibited higher ductility at failure than did the T-beam. The displacements of the triangular beams I and II at collapse were approximately 26 and 21 mm, respectively. On the other hand, the displacement of the T-beam at collapse was approximately 17 mm. This shows that the plastic deformation at failure of the triangular beams was higher than that of the T-beam. This is a strong indication of the higher ductility of the triangular beams compared to the T-beam.

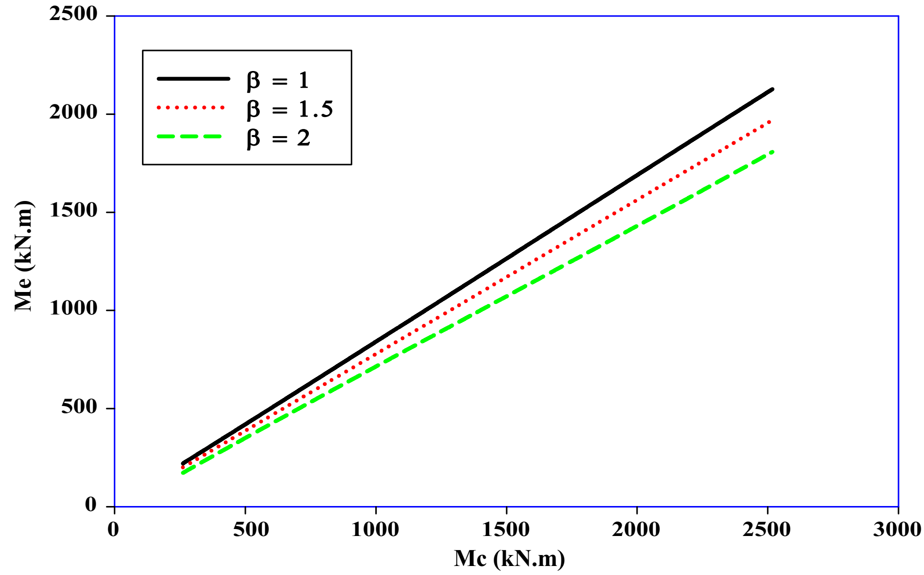

Fig. 4 Reliability index of T-beams with neutral axis in the web

Fig. 5 Instron HDX1500 static universal testing system

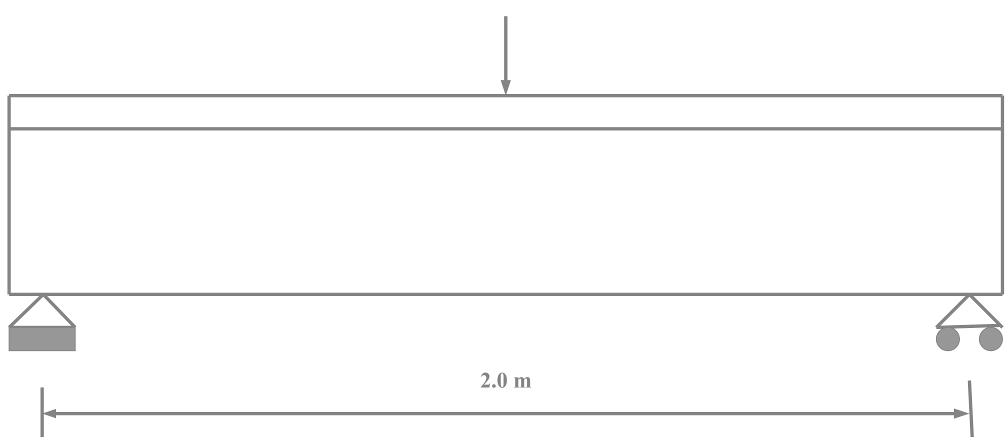

Fig. 6 Test setup schematic representation

Table 4 Experimental beam dimensions

Beam type	b_w (mm)	b_f (mm)	t_f (mm)	d (mm)	h (mm)	Gross area (mm^2)
T-beam	150	350	50	170	200	40,000
Triangular I	250	–	–	187	217	27,125
Triangular II	300	–	–	230	260	39,000

Table 5 Concrete mix design

	Coarse aggregates (kg)	Fine aggregates (kg)	Cement (kg)	Water (kg)
Weight per m^3 of concrete	1076	709	348	201

Table 6 Testing machine output results

Time (s)	Extension (mm)	Strain (%)	Load (N)	Flexure stress (MPa)	Flexure extension (mm)	Flexure strain (%)	Flexure load (N)	Displacement (m)	Corrected position (mm)
1.1	0.002	3.47E−06	387.1283	−0.11769	−0.004	−0.00017	−387.1283	6.94E−06	0.002
1.2	0.002	5.96E−05	386.4234	−0.11747	−0.004	−0.00017	−386.4234	1.19E−04	0.002
0.8	−0.001	2.84E−05	383.9681	−0.11673	−0.001	−4.16E−05	−383.9681	5.69E−05	−0.001
1.8	−0.002	1.04E−06	383.6979	−0.11664	−3.26E−19	−1.36E−20	−383.6979	2.09E−06	−0.001
0.7	0.000	4.12E−05	382.4292	−0.11626	−0.002	−8.31E−05	−382.4292	8.25E−05	0.000
1.5	−0.001	−2.47E−06	382.0063	−0.11613	−0.001	−4.16E−05	−382.0063	−4.94E−06	−0.001
0.5	−0.001	−6.55E−05	381.9828	−0.11612	−0.001	−4.16E−05	−381.9828	−1.31E−04	−0.001
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494.8	−15.125	3.71E−03	−41075.9900	12.48710	15.123	0.628380	41075.99	7.41E−03	−15.125
493.6	−14.889	3.67E−03	−41089.5000	12.49121	14.887	0.618730	41089.50	7.34E−03	−14.889
494.2	−15.012	3.70E−03	−41098.5400	12.49395	15.010	0.623684	41098.54	7.40E−03	−15.012
495.1	−15.179	3.68E−03	−41119.8400	12.50030	15.177	0.630623	41119.41	7.36E−03	−15.179
494.1	−14.993	3.72E−03	−41124.8700	12.50196	14.991	0.622895	41124.87	7.41E−03	−14.993
494.7	−15.106	3.65E−03	−41125.1500	12.50204	15.104	0.627590	41124.15	7.31E−03	−15.106
495.2	−15.198	3.78E−03	−41129.3500	12.50332	15.196	0.631413	41129.35	7.56E−03	−15.198
494.6	−15.089	3.74E−03	−41130.0800	12.50354	15.087	0.626884	41120.08	7.47E−03	−15.089

The simple beam moment formula $\frac{PL}{4}$ was used to compute the external moment M_e for all the beams. The computed values of M_e for Triangular I, Triangular II, and T-beams were 17, 22.5, and 20.5 kN m, respectively. Equations 4, 10, and 13 were used to compute the reliability index β of the experimental beams. Figure 9 shows the variation of the reliability index β with respect to the external moment M_e. As shown in the figure, the values of the reliability index β for the three beams were either negative or very close to zero at their respective collapse load.

Table 8 summarizes the flexural crack width results during beam testing. The results show that flexural crack width for the three beams was large ranging from 7 to 10 mm for the triangular I beam, 5 mm to 7 mm for the triangular II beam, and 6 to 11 mm for the T-beam.

Finite element analysis

A nonlinear finite element analysis was conducted using the commercial software STAAD-PRO V8i., to simulate the experimental beams shown in Fig. 10 (Zhang 2013; Yazdizadeh 2013; Rao and Rao 2012). Figures 11 and 12 shows the structural models of the triangular and T-beams, respectively. Solid and beam elements were used to model the concrete material and reinforcing steel bars, respectively. Geometric rather than material non-linearity (second-order analysis) was considered in this study. The failure criteria considered herein is the displacement.

Table 9 and Fig. 13 show the displacements of the three beams obtained using the finite element model and those obtained experimentally. The results show that the finite element vertical displacements compared well with those

Fig. 7 Crack pattern and shape of triangular and T-beams after collapse

Table 7 Beam collapse load and deflection

Beam type	Collapse load (kN)	Deflection (mm)
T-beam	41	15
Triangular I	34	18
Triangular II	45	24

obtained experimentally. It is worth noting that horizontal and buckling displacements in the finite element model were set equal to 2.0 mm and zero, respectively. The results also show that the displacements obtained using the finite element model were larger than those obtained experimentally. In other words, the displacements obtained using the finite element method were more conservative than those obtained experimentally. This is an indication that the finite element method can provide reliable analysis results comparable to those obtained experimentally.

Discussion

Three reinforced concrete beams were tested experimentally and analyzed analytically using the finite element method. Their reliability was also assessed using the reliability index approach. The safety percentage values for the triangular beams were equal to 15, 20, and 26 % for β values of 1, 1.5 and 2, respectively. On the other hand, the safety percentage values for the T-beam with the neutral axis in the flange were equal to 15, 21 and 27 % for β values of 1, 1.5 and 2, respectively. The safety percentage values for the T-beam with the neutral axis in the web were

Fig. 8 Beam load deflection response

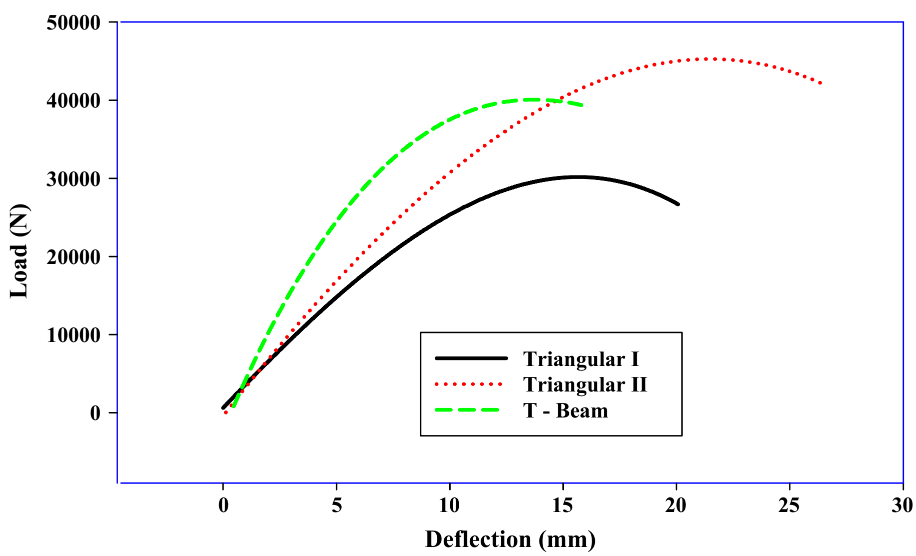


Fig. 9 Reliability index versus external moment for experimental beams

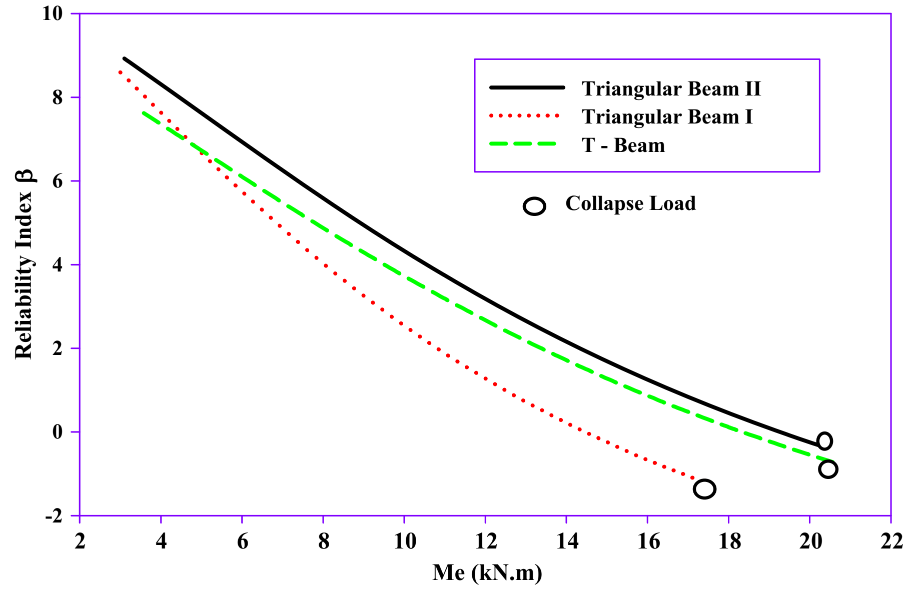
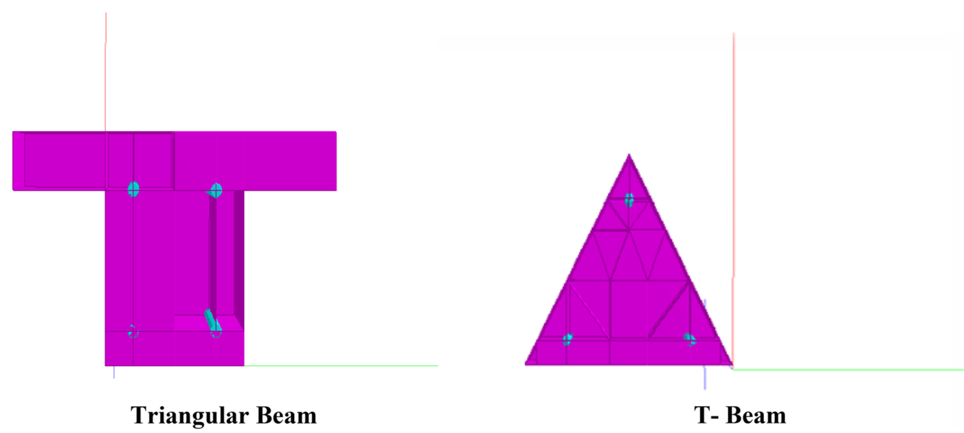


Table 8 Experimental beam cracking load and width

Beam type	1st crack		2nd crack		3rd crack	
	Load (kN)	Width (mm)	Load (kN)	Width (mm)	Load (kN)	Width (mm)
Triangular I	15.4	7	16.9	9	21.3	10
Triangular II	20.7	5	23.1	6	26.2	7
T-beam	13.6	6	14.9	8	19.1	11

Fig. 10 Finite element solid models



equal to 16, 22, and 28 % for β equals 1, 1.5 and 2, respectively. The experimental load–deflection response of the beams showed that the triangular beams had a better toughness and exhibited higher plastic deformation before failure. This is an indication that a T-beam would collapse faster than a triangular beam for an equal area of steel and concrete. The 1st, 2nd, and 3rd cracks showed that the Triangular-II beam exhibited smaller cracks than did the T-beam for larger loads even though it had a smaller concrete gross area. The design moment strength M_c that was computed using the experimental collapse loads were 20.5, 17 and 22.5 kN m for the T-beam, Triangular-I beam, and Triangular-II beam, respectively. The design moment strength M_c computed using the ACI design code were 18.35, 14.45, and 21.6 kN m for the T-beam, Triangular-I beam, and Triangular-II beam, respectively. The reliability analysis of the experimental data predicted a low reliability index β of -0.6, -1.0 and -0.4 at the collapse load for the T-beam, Triangular-I and Triangular-II, respectively. The finite element beam displacement at the collapse load was

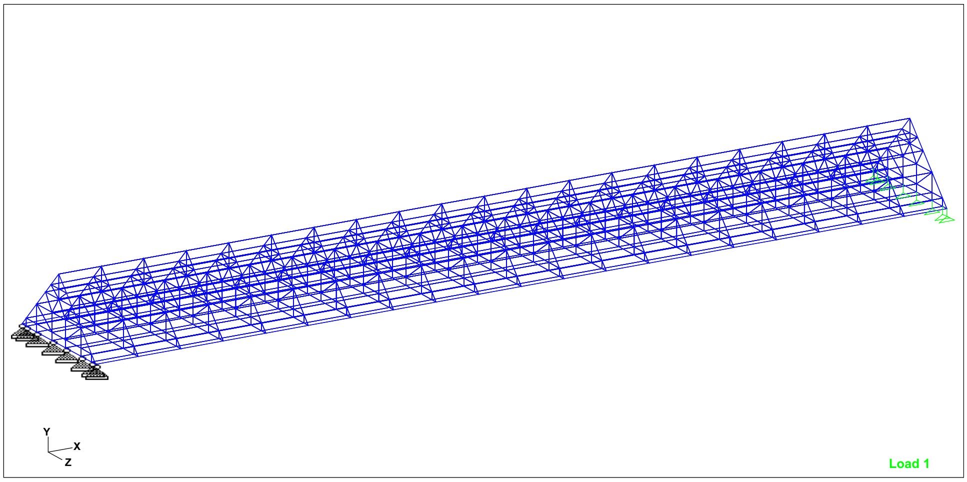


Fig. 11 Triangular beam structural model

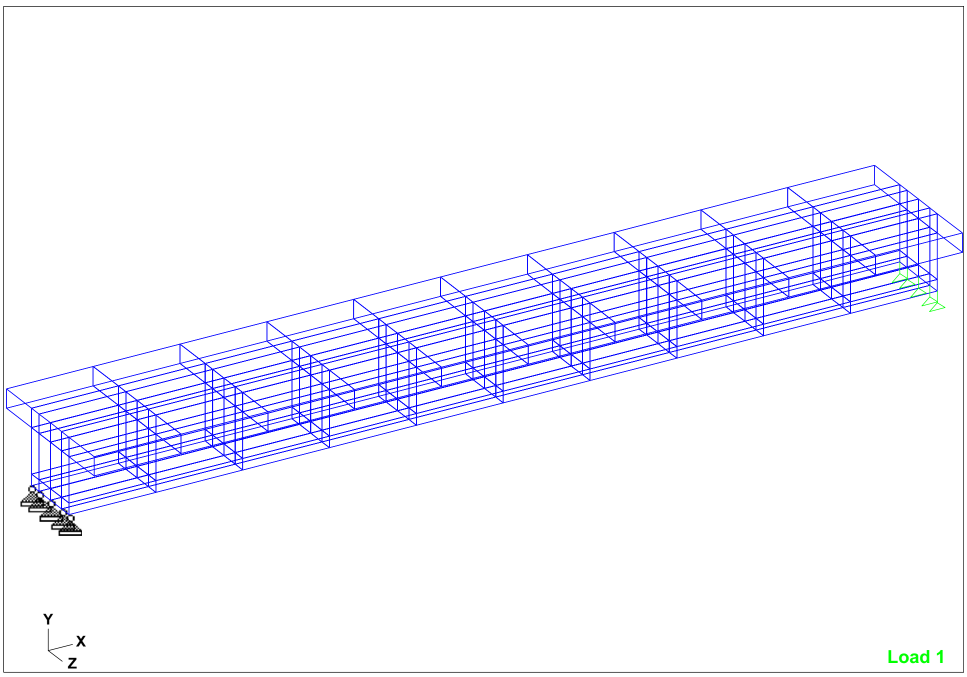


Fig. 12 T-beam structural model

Table 9 Beam finite element model and experimental displacements

Beam type	Collapse load (kN)	Finite element dispacements			Experimental displacement
		δ_{HFE} (mm)	δ_{VFE} (mm)	δ_{ZFE} (mm)	δ_{VEXP} (mm)
T-beam	41	2	17	0	15
Triangular-I	34	2	24	0	18
Triangular-II	45	2	26	0	24

larger than the experimental one by 11, 25, and 7 % for the T-beam, Triangular-I, and Triangular-II, respectively. These percentages indicate that the finite element nonlinear analysis is safe and yields results that are close to those obtained experimentally, provided that the maximum allowed horizontal displacement is limited to 2.0 mm and no buckling displacement is allowed.

Conclusions

The paper studied the behavior of triangular and T-reinforced concrete beams. Numerical and experimental studies were conducted to study the behavior of simply supported triangular and T-beams. Their reliability was also assessed using the reliability index approach. The

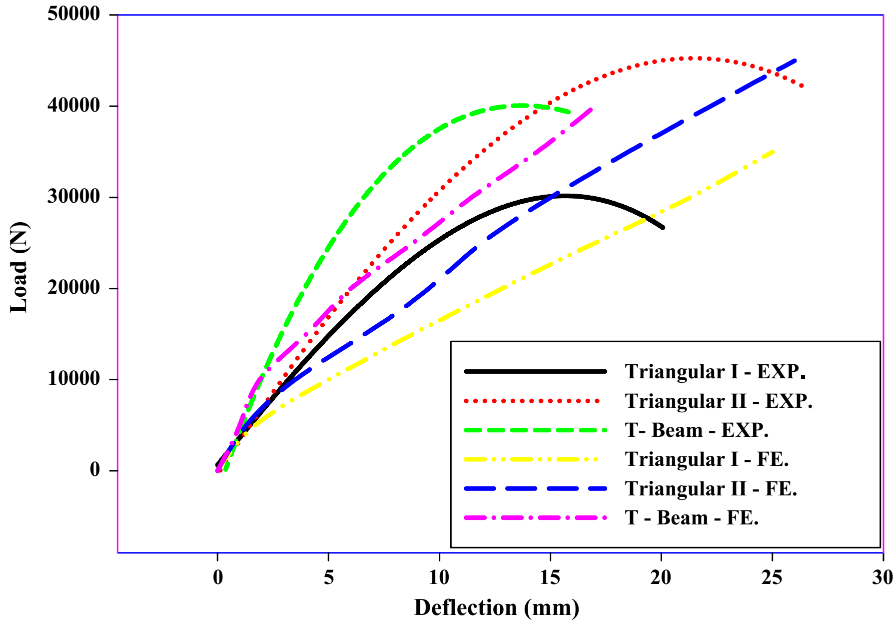


Fig. 13 Experimental and finite element beam responses

results showed that the vertical displacements that were computed using the finite element method compared well with those obtained experimentally. The difference between the displacements obtained using the finite element model and those obtained experimentally was larger than 8 %. In other words, the displacements obtained using the finite element analysis were conservative. This is an indication that the finite element method can provide reliable analysis results comparable to those obtained experimentally. The results showed that the triangular beams exhibited higher ductility at failure than did the T-beam. The displacements of the triangular beams I and II at collapse were approximately 26 and 21 mm, respectively. On the other hand, the displacement of the T-beam at collapse was approximately 17 mm. This shows that the plastic deformations at failure of the triangular beams were higher than that of the T-beam. This is a strong indication of the higher ductility of the triangular beams compared to the T-beam. Triangular beams have smaller cracks under large loads than do T-beams for equal areas of steel and concrete. The design moment strengths M_c computed using the ACI design code formulation were safe and close to those computed using experimental results. The experimental results verified the assessment of the reliability analysis that stated that the triangular beams are more reliable than T-beams for equal areas of steel and concrete.

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