# EARTHQUAKE RESPONSE OF STRUCTURES BY STRUCTURAL MIXTURE THEORY

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**ABSTRACT:** A structural mixture theory is developed for use in predicting the response of a structural frame to earthquake ground motion. The frame is considered to be a mixture of columns and beams and is modeled as two interacting subsystems. The dynamic response of each subsystem is described by its own linear differential operator. The two responses are then subjected to matching conditions (i.e., boundary conditions) which couple them at the subsystem interface. The conditions include damping ratios, moment-of-inertia ratios, frequency ratios, and the effects of relative node/support motion. The solution of the coupled equations is carried out by means of a pair of coupled perturbation series. Example results from the numerical implementation of the theory are given, and compared with the result of the corresponding calculation using the Wilson- $\theta$  method and observed earthquake response. The structural mixture theory is valid for multiple-degree-of-freedom structures under the type of ground motions used in this study, and it appears to be applicable to systems with linear or nonlinear behavior.

## INTRODUCTION

### **Earthquake Ground Motion**

The occurrence of earthquakes and their consequent impact on people and the facilities they live and work in has been with us since the early ages of the earth's formation. On the average 10,000 people die each year as a result of earthquakes (Kanamori 1978), mainly through the collapse of buildings that cannot withstand the characteristic shaking motion. Earthquakes are of special interest in structural engineering because they are the only natural phenomena that can affect, almost simultaneously, all components of a structural system. They occur at irregular time intervals and transmit transient dynamic ground motions and forces, which are among the largest that can be applied to all types of structures.

Earthquake ground motion is usually measured by a strong motion accelerograph which records the acceleration of the ground at a particular location. The recorded accelerograms, after they are corrected and adjusted, may be integrated to obtain velocity and displacement time histories. Some records of seismic events, such as the San Fernando (1971) and El Centro (1940) earthquakes, are frequently used in earthquake engineering. Characteristics that are important in the seismic analysis of earthquake engineering design include: (1) the peak ground motion values (acceleration, velocity, and displacement); (2) the time duration of strong ground motion; and (3)the frequency content (obtained through Fourier analysis). The state of the art in strong motion seismology and earthquake engineering has advanced considerably in the last two decades, in conjunction with the strengthening of California building codes following the San Fernando earthquake (Bolt 1988). It is often possible to estimate reasonably the peak ground motions, durations, and spectral shapes expected at a given site.

## **Earthquake Response of Structures**

Structural response to seismic motions is complex. It is interdependently affected by the characteristics of the input mo-

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tion at the base of the structure, the stiffness characteristics of the foundation materials, and the dynamic response characteristics of the structure. The structure's dynamic characteristics include its mode shapes and natural vibration periods. These in turn are functions of the distribution of mass and stiffness between the sections of supporting columns and the adjacent floors.

The development of analytical methods for predicting the response of structures has been one of the principal aims of earthquake engineering. The earliest analyses were based on a single-degree-of-freedom (SDOF) model that expresses the response in terms of a single displacement coordinate (at the top of the structure). This continues to be the basis for most preliminary design estimates of the stresses resulting from earthquake loading.

Matrix formulation of the response equations has been a major step in the development of procedures for more complex dynamic analysis, because it makes it possible to deal with multiple-degree-of-freedom (MDOF) structural models. Dynamic analysis techniques were formulated in parallel with matrix methods for static structural analysis, and both were concurrent with the development of digital computers which can handle the vast quantities of numerical operations required for the solution of MDOF systems. For more details on the equations of motion and their simultaneous solution (Paz 1991; Clough 1975; Craig 1981). These methods basically involve the modal decomposition of linear systems of equations.

The determination of the nonlinear response of MDOF structural models requires a step-by-step numerical integration of the equations of motion (in time). One of the simplest of the many methods available is a modification of the linearacceleration method, known as the Wilson- $\theta$  method (Wilson et al. 1973). The Wilson- $\theta$  method is unconditionally stable, i.e., numerical errors do not tend to accumulate during the integration process, regardless of the selected magnitude of the time step. It is also equally applicable to systems with linear or nonlinear behavior, and is therefore quite convenient to use as a benchmark with which other approaches to the calculation of structural earthquake response may be compared. For application of the Wilson- $\theta$  method to the earthquake response of fixed offshore structures, see Kirkely (1973).

#### **Structural Mixture Theory**

In this paper we present a new approach to the calculation of the earthquake response of structures, based on a structural mixture theory. Originally, the concepts and methods of this type of theory were developed for mixtures of two continua, such as sand and seawater, or water in a saturated porous solid

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