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Cost of Reinforced Concrete Paraboloid Shell Footing

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ABSTRACT

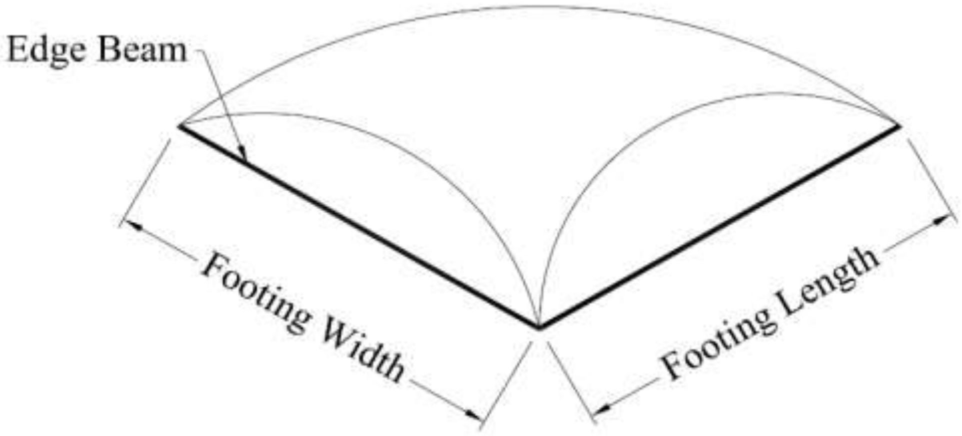
The shell footings with different flatness ratio were analyzed and designed optimally to ACI code of design in order to minimize the total cost of the footing material that includes cost of concrete, cost of steel, and formwork cost. Shell crown displacement under the load is very important factor of safety in shell footing design. Shell footings displacements with different flatness ratio were small in magnitude and the best among them are the ones with high flatness ratios. The cost of shell footings with different flatness ratios compared well with the cost of conventional square footing with flatness ratio zero and it is more economical than conventional square footing in most cases for footings in poor soil. The process of computing optimized shell footing material cost presented in this paper is flexible and could be used for other codes of design by modifying optimization equations to estimate shell footings material cost. Numerical examples are presented to illustrate the validity of the process of computing optimized shell footing material cost for a desired axial load.

Keywords: Shell footing, Flatness ratio, Concrete, Steel, Formwork, Optimization, Material cost.

INTRODUCTION

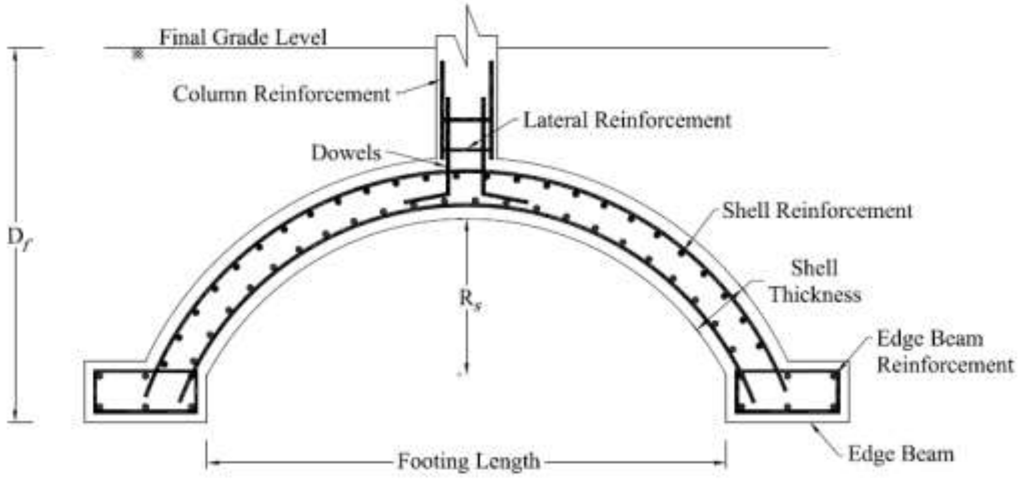
Shell is a curved structural element in which the thickness is small compared to the lateral dimensions and radii of curvature. Reinforced concrete shell footings have been increasingly used for columns transmitting heavy loads to weak soils. Conical and hyperbolic paraboloid shells behavior as footing have been studied through experimental testing and finite element analysis and it proves to be more efficient than conventional flat footing (1 and 2). Other shell geometry such as elliptical paraboloid shell could be used as isolated shell footing clamped with edge beams, Fig. 1, (3).

Fig. 1 Paraboloid Shell Clamped with Edge Beams



Safety and reliability were used in the flexural design of reinforced concrete shell footings using ultimate-strength design method USD under the provisions of ACI building code of design (4). Shell footing sizes are mostly governed by the axial load P, allowable soil pressure Qa, unit weight of concrete **γc**, soil unit weight **γs**, and the depth of the footing base below the final grade $\boldsymbol{D_f}$. The optimized dimensions of reinforced concrete shell footing could be achieved by minimizing the optimization function of shell thickness and reinforcing steel area, Fig.2, (5).

Fig. 2 Shell Footing Dimensions and Reinforcement Detailing



This paper presents a process of computing optimized shell footing material cost for column axial load and desired flatness ratio ***r*** that is the ratio of shell rise $\mathbf{R_S}$ to shell footing length **L**. The optimization of footings is formulated to achieve the best footing dimension that will give the most economical section to resist the external axial loads **P** that is made of summation of dead loads **DL** and live loads **LL** for different flatness ratios. The optimization is subjected to the design constraints of the building code of design ACI such as maximum and minimum reinforcing steel area, footing depth, developmental length in tension and compression, (6).

The total cost of the footing materials is equal to the summation of the cost of concrete, steel and formwork. The required footing area F_A is computed based on the axial load P and the effective soil pressure Qe:

$$F_A = \frac{P}{Qe} = \frac{DL+LL}{Qe} \qquad (1)$$

$$Qe = Qa - Wc - Ws \qquad (1A)$$

$$Wc = \gamma c * h_S \qquad (1B)$$

$$Ws = \gamma s * D_f \qquad (1C)$$

Where

F_A = Footing area

Qa = Allowable soil pressure

Qe = Effective soil pressure

Wc = Concrete weight

Wc = Soil weight

h_S = Total shell footing thickness

Finite Element Analysis

A finite element structural model was used to simulate the paraboloid shell footing, Fig. 3, (7).

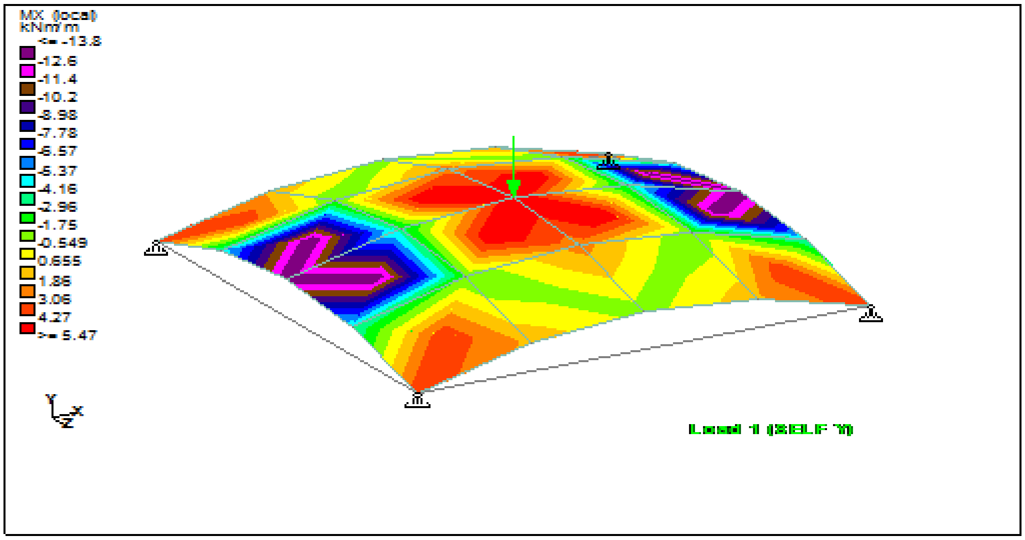


Fig. 3 **Paraboloid Shell Footing Structural Model**

The values of shell crown vertical displacement $\boldsymbol{\delta_V}$, transverse and longitudinal moments in x plane and z plane respectively for different values of flatness ratio $\boldsymbol{r}$ and shell design parameters are computed for allowable soil Pressure $\boldsymbol{Qa}$ of 50 KPa, Specified yield strength of nonprestressed reinforcing $\boldsymbol{fy}$ of 420 MPa, concrete unit weight $\gamma\mathbf{c}$ of 25 kN/m^3, soil unit weight $\gamma\mathbf{s}$ of 18 kN/m^3, 1 m depth of the footing base below the final grade D_f and specified compression strength of concrete $\boldsymbol{f`c}$ of 30MPa, Table 1, Fig. 4, (8 and 9).

Table 1 Shell Moments and Vertical Displacements

Size m X m	$r = \frac{S_R}{L}$	Thickness mm	Mx $\frac{kN.m}{m}$		M_Z $\frac{kN.m}{m}$		δ_V mm
			Top	Bottom	Top	Bottom	
2 X 2	1/4	300	1.1	5.1	1.1	5.1	0.15
3 X 3	1/7		1.5	3.2	1.5	3.2	1.0
2 X 4.5	1/7		1.4	6.3	1.9	4	6.0
4 X 4	1/5		1.8	8.4	1.8	8.4	1.0
5.5 X 5.5	1/6		1.1	2.5	1.1	2.5	2.49

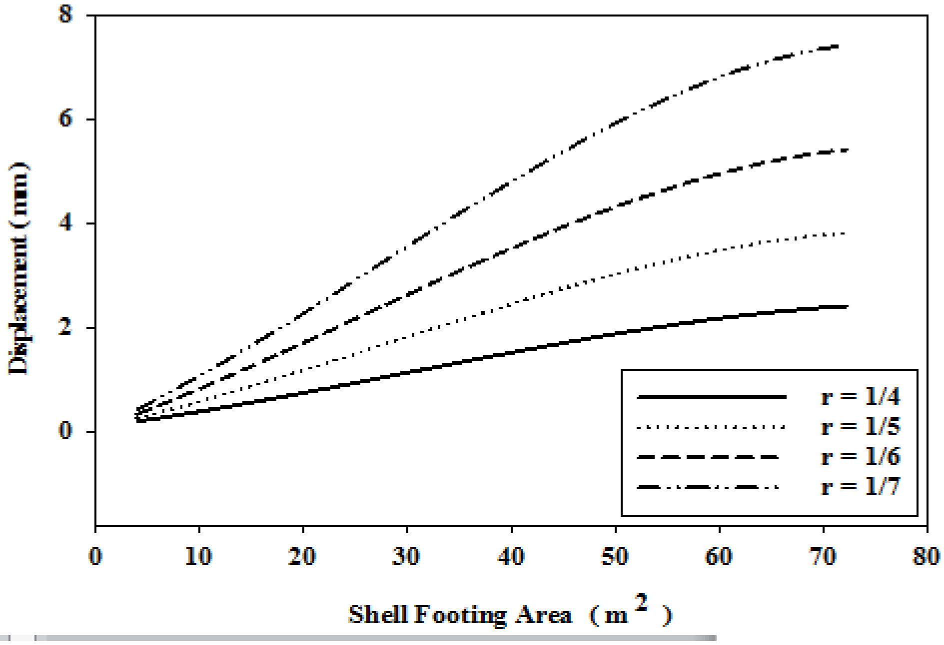


Fig. 4 Displacement of a Square Base Paraboloid Shell Footing

Vertical displacements δ_V under the column axial load and longitudinal and transverse moments top and bottom control the shell footing design. Other stresses, moments and displacements are neglected because of their little contribution if any to the shell footing design. It is obvious that a square shell base is more economical than a rectangular base shell. The 3X3 shell footing has an area of 9m^2, displacement of 1mm and a max moment of 3.2kN.m/m, on the other hand a 2X4.5 shell footing has an area of 9m^2, displacement of 6mm and a max moment of 6.3kN.m/m. Therefore a shell footing with square base will be optimized.

Footing Optimization

The optimization of shell footing is formulated to achieve the best footing dimensions that will give the most economical shell footing size with square base and steel reinforcement to resist the longitudinal and transverse Top moments M_T and bottom moments M_B. The optimization is subjected to the constraints of the building code of design ACI for depth, reinforcement and footing size. The optimization function of the shell footing with square base

Minimize $F(As, d) = \varphi_b\, As\, fy\left(d - \frac{a}{2}\right) - M_T$ (2-A)

Minimize $F(As, d) = \varphi_b\, As\, fy\left(d - \frac{a}{2}\right) - M_B$ (2-B)

The optimization function of the edge beam that is clamped to the shell footing

Minimize $F(As, d) = \varphi_b\, As\, fy\left(d - \frac{a}{2}\right) - M_{EBP}$ (3-A)

Minimize $F(As, d) = \varphi_b\, As\, fy\left(d - \frac{a}{2}\right) - M_{EBN}$ (3-B)

Where

φ_b= Bending reduction factor

fy = Specified yield strength of nonprestressed reinforcing

As = Area of steel

d = Effective depth

a =Depth of the compression block

M_{EBP} = Edge beam positive moment

M_{EBP} = Edge beam negative moment

Must satisfy the following constraints:

$d_S^L \leq d \leq d_S^U$ (4-A)

$As_S^{Mini} \leq As \leq As_S^{Max}$ (4-B)

$\delta_V \leq 10$ mm (4-C)

$As^{Max} = 0.75 * \beta 1 * \frac{f`c}{fy}\left(\frac{600}{600+fy}\right) bd$ (4-D)
$As^{Mini}\, As_{EB}^{Mini} = \left(\frac{1.4}{fy}\right) bd$ (4-E)
$As_S^{Mini} = 0.002 b h_S$ (4-F)
$b = 1$ meter strip width
$\beta 1 = 0.85 \;\; for \;\; f`c \leq 30\, MPa$ (4-G)
$\beta 1 = 0.85 - 0.008(f`c - 30) \geq 0.65 \;\; for \;\; f`c > 30\, MPa$ (4-H) $h_S \geq 300\, mm$
(4-I)
$h_{EB} \geq \frac{L}{21}$ (4-J)
h_{EB} = Edge beam thickness
L = Edge beam length

Where d_B^L and d_B^L are shell footing and edge beam depth lower and upper bounds, and As_B^{Mini} and As_B^{Max} are shell footing and edge beam steel reinforcement area lower and upper bounds. The reinforcing bars must have the required length to provide sufficient strength. In other words, the bars must extend developmental length Ld in the shell footing.

For the dowel bars under compression (ACI Section 12.3)

$$As_{dowels} \geq 0.005\, A_{Column} \qquad (4\text{-}K)$$

Where
As_{dowels} = Steel area of the dowels
A_{Column} = Column area

Shell Footing Formwork Materials

The formwork material of the shell and edge beam is timber. Beam formwork consists of beam bottom 50 mm thickness and two sides of 20mm thick plywood. For the shell footing a cubic meter of concrete requires 0.2 m^3 of timber forms that are made up of timber battens lined with plywood, (10 and 11).

Shell Footing Cost Analysis

The total cost of the shell footing materials is equal to the summation of the cost of the concrete, steel and timber:

$$Total\ Cost = CV(m^3) * Cc + SV(m^3) * \gamma_s \left(\frac{Ton}{m^3}\right) * Cs + TV(m^3) \qquad (6)$$

Where
CV = Concrete volume of shell and edge beam
SV = Steel volume of shell and edge beam
TV = Timber volume of shell and edge beam
Cc = Cost of 1 m^3 of ready mix reinforced concrete in dollars
Cs = Cost of 1 Ton of steel in dollars
Cf = Cost of 1 m^3timber in dollars
γ_s = Steel density = $7.843 \frac{Ton}{m^3}$

Optimized design results showed that minimum thickness of concrete and minimum area of steel is sufficient for economical and safe design of paraboloid shell footing. The cost of shell footing materials for different flatness ratios is computed based on Qatar and USA prices respectively of \$100,\$131 for 1m^3 of ready mix concrete, 1170,\$1100 for 1 ton of reinforcing steel bars, and \$531, \$565 for 1m^3 of timber,(12). Shell footings with flatness ratio r of $\frac{1}{4}$ are the most economical with respect to concrete cost. Shell footings with flatness ratio r of $\frac{1}{7}$ are the most economical with respect to steel and timber cost. Table#, Fig. ##, (13).

Table 2 Shell Footing Material Cost $

Size mXm	Material	USA Cost $			
		$r = \frac{1}{4}$	$r = \frac{1}{5}$	$r = \frac{1}{6}$	$r = \frac{1}{7}$
2X2	Concrete	123	152	147	144
	Steel	78	76	77	71
	Timber	130	132	129	127
4X4	Concrete	524	650	629	616
5X5	Steel	484	469	524	441

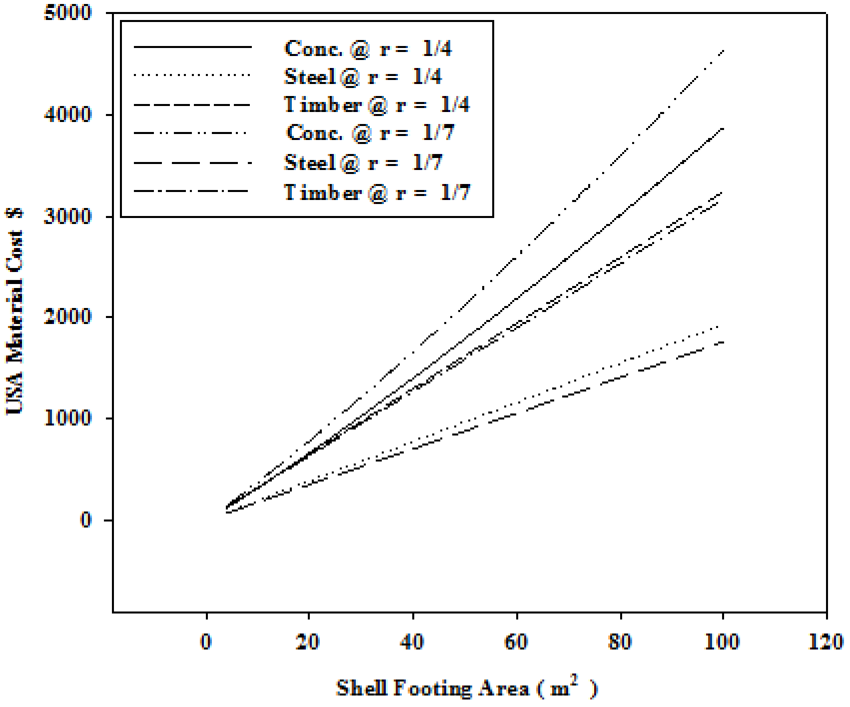


Fig. 5 Material Cost of a Paraboloid Shell Footing

RESULT AND DISCUSSION

The shell footings with different flatness ratio were analyzed and designed optimally to ACI code of design in order to minimize the total cost of the footing that includes cost of concrete, cost of steel, and formwork cost. The footings were sized based on column axial load and effective soil pressure Q_e. In order to optimize the footing thickness and steel area for both shell footing and edge beam, a list of constraints (equations 4A-4K) such as vertical displacement under the axial load, shell footing and edge beam area of steel and concrete thickness and dowels developmental length have to be met. Volumes of concrete CV, reinforcing steel SV and timber TV are computed based on optimum footing dimensions. The total cost of footing material is calculated using equation 5 based on Qatar and USA prices respectively of $100,$131 for 1 m^3 of ready mix concrete, $1070,$1100 for 1 ton of reinforcing steel bars, and $531, 565 for 1 m^3 of timber. Shell footing with flatness ratio ***r*** ranging from $\frac{1}{7}$ to $\frac{1}{4}$ showed that high flatness ratio $\frac{1}{4}$ yielded a small displacement and economical quantities of concrete. Shell footing with Low flatness ratio $\frac{1}{7}$ yielded smaller quantities of steel and timber than shell footing with higher flatness ratio, but it has bigger displacement and bigger concrete quantities when compared with shell footing with higher flatness ratio. The cost of Shell footings with flatness ratio $\frac{1}{4}$ compared well with flat square footing with flatness ratio zero, that is the most common and economical type of footings. For footings of areas bigger than 20 m^2 the shell footing is more economical and cost less than square footing. Both types of footings cost about the same for footings of areas less than 20 m^2 , Table 3 , Fig. 6.

Table 3 Shell Footing Total Material Cost $

Area m^2	USA Total Cost $				
	$r = 0$	$r = \frac{1}{4}$	$r = \frac{1}{5}$	$r = \frac{1}{6}$	$r = \frac{1}{7}$
4	511	334	360	352	342
9	877	767	821	810	780
16	1378	1388	1479	1470	1406
25	2681	2209	2343	2345	2229
36	3644	3238	3421	3446	3256
49	5989	4485	4720	4784	4495
64	7563	5960	6247	6371	5954
81	10290	7672	8012	8218	7641
100	13590	9630	10020	10340	9563

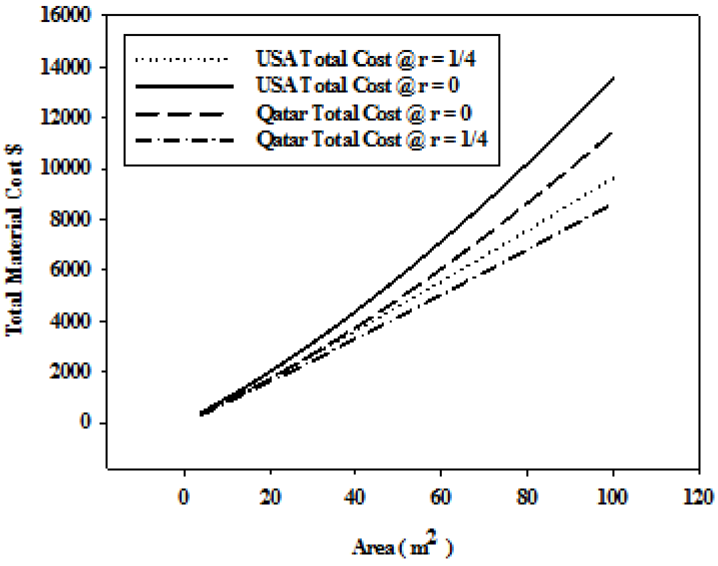


Fig. 6 Total Material Cost of Footing

The design parameter used in estimating the footing material cost based on optimal criteria are 400 MPa, 30 MPa, 50KPa, 25kN/m^3 , 18kN/m^3 and 1meter for ***fy, f`c, Qa, γc, γs*** and D_f respectively for a column ultimate axial load ranging from 100kN to 3000kN as the maximum axial load. In fact shell footings could safely carry the column load with shell thickness less than the required minimum thickness by ACI code of design. More economical cost of shell footings with thickness of 250mm and 200 mm would have been estimated by the process of computing optimized shell footing cost if the codes of design allow such thicknesses, Fig. 7, (14).

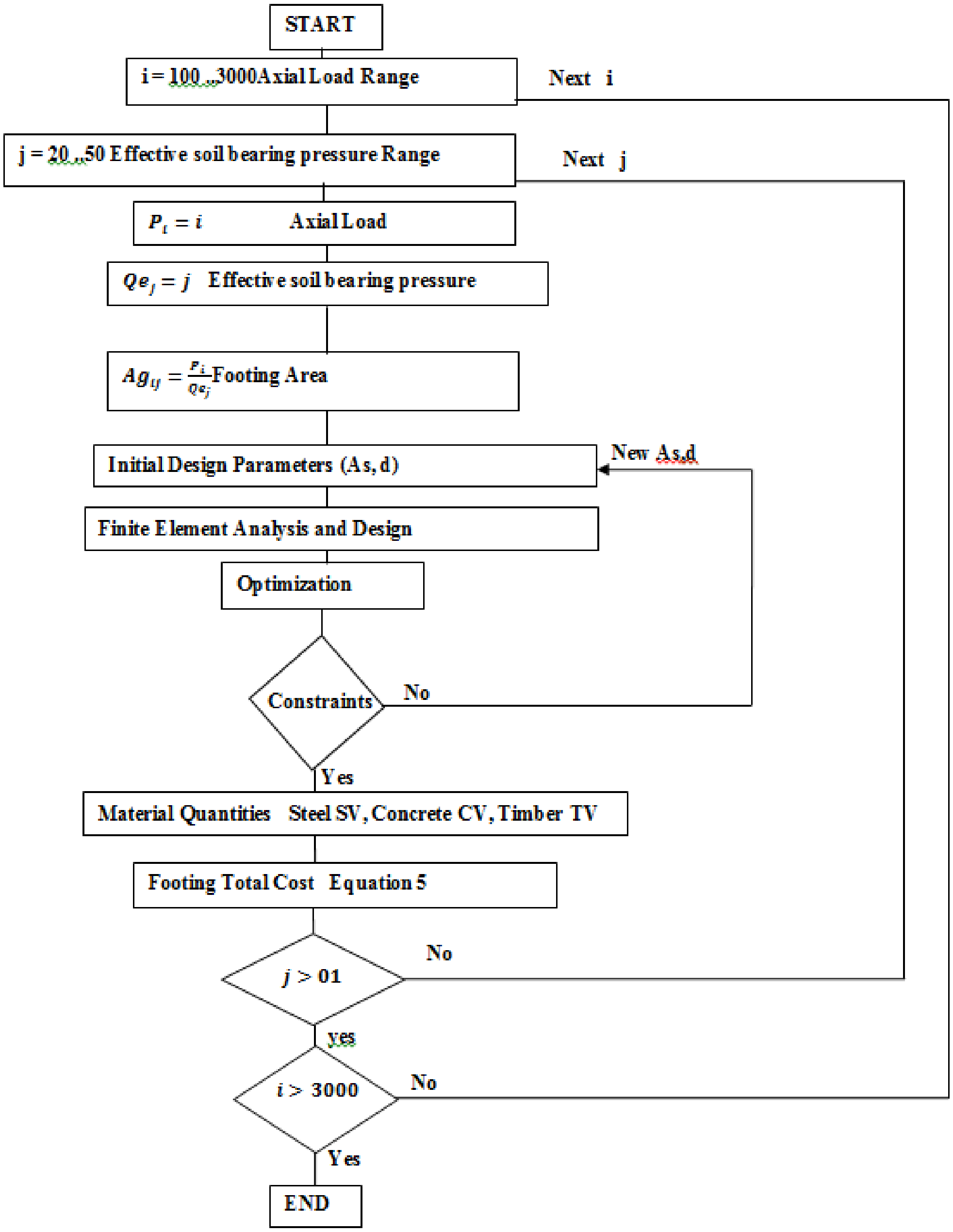


Fig. 7 The Process of Computing Optmized Shell Footing Cost

CONCLUSIONS

Optimum design of shell footing with high flatness ratio yielded a small displacement and economical quantities of concrete. Shell footing with Low flatness ratio yielded smaller quantities of steel and timber than shell footing with higher flatness ratio, but it has bigger displacement and bigger concrete quantities when compared with shell footing with higher flatness ratio. Shell crown displacement under the load is very important factor of safety in shell footing design; therefore shell footing with high flatness ratio is safer, economical and practical. Displacements of shell footing crown under the axial load is very low, so the shell footing thickness is limited to the minimum requirements by the code of design, and it leads to a low settlement of the shell footings corners that is much lower than the allowed settlement. Square shell footing is more economical and it has smaller displacement than rectangular shell footing and because of symmetry the square shell footing has vertical displacement only under the axial column load and equal transverse and longitudinal moments top and bottom of the shell crest. All shell footings with low and high flatness ratios required minimum area of steel and minimum concrete thickness based on ACI code of design. In fact shell footings could safely carry the column load with shell thickness less than the required minimum thickness by ACI code of design. The cost of shell footings with different flatness ratios compared well with the cost of conventional flat square footing with flatness ratio zero, that is the most common and economical type of footings. For footings of areas bigger than 20 m^2 the shell footing is more economical and cost less than square footing. Both types of footings cost about the same for footings of areas less than 20 m^2 , The process of computing optimized shell footing cost presented in this paper is flexible and could be used for other codes of design by modifying optimization equations to estimate shell footings cost.

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